The Chain Needed Is Not Proportional to the Depth

— The shortest chain that lets the anchor be pulled horizontally is $\sqrt{d^2 + 2da}$, with a the load divided by the chain's submerged weight, so its ratio to the depth depends on a/d alone — at a fixed load the ratio falls with depth, to **1.500000** at 80 m — [Paper 568]

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September 2026

— *From thinking to syncing.* —

Abstract

When anchoring, chain is paid out as a multiple of the depth.What this paper shows is that the ratio of chain to depth is not constant but set by a/d alone, and that what keeps the anchor pulled horizontally is not the anchor but the weight of the chain.No new theorem or law is claimed. First, the length needed has a closed form——the shortest chain meeting the seabed at the anchor is $\sqrt{d^2 + 2da}$, its shape the catenary of Paper 327 (Section 2). **Second, and this is the core.The ratio to depth depends on a/d alone——"five to seven times" falls exactly at $a/d = 12$ and 24, returning 5.000000 and 7.000000** (Section 2). **Third, at a fixed load the ratio falls with depth——4.582576 at 5 m and 1.500000 at 80 m, though the length itself grows** (Section 3). **Fourth, doubling the load does not double the chain——1.381699 times at a depth of 10 m** (Section 3). **Fifth, a chain too short lifts the anchor——1.960175 degrees at 0.95 of the length needed, 30.823826 at half** (Section 4). **Sixth, the separator is whether the line has submerged weight——a weightless line pulls at 11.536959 degrees even at a scope of five, and no length makes it horizontal** (Section 5). **Keywords**: catenary, anchor chain, mooring, water depth, holding power.

Scope of this paper (scope note): **No new theorem or law is claimed——the catenary and the statics of mooring lines are standard.No anchoring practice is recommended——how much chain to pay out is not stated; only what a given length does.Anchor holding capacity is excluded——how much force an anchor withstands is an empirical matter of seabed and anchor form, and it is not included; only the geometry of the angle at which the anchor is pulled is examined.The chain is inextensible and uniform, the seabed flat——neither chain elasticity nor slope is included.Dynamic tension is not treated——fluctuations from waves, swell and vessel motion are not the subject.The height of the bow above water is excluded——including it would add to the depth d.The load is given——it is not computed from wind or current; a is supplied as the load divided by the chain's submerged weight.The rule of thumb is not rejected——only the range of a/d in which it holds is shown.Relation to earlier papers**: **Paper 327 showed that two curves agreeing to within 0.083 per cent are still not the same curve, a hanging chain being a catenary because its load is counted along the arc——this paper uses that curve to ask how long the chain must be: that one separates shapes, this one measures a length.Paper 556 showed that what carries most of an external load is not the bolt——the same shape here: what pulls the anchor horizontally is not the anchor but the weight of the chain, the inconspicuous part doing the work.Paper 300 showed that sharing a root is decidable——"the ratio depends on a/d" and "a short chain lifts the anchor" share one root in the catenary.**What is added is **confirming to 12 digits that the scope depends on a/d alone, by two matched pairs of depth and load, showing that "five to seven" falls exactly at $a/d = 12$ and 24, solving by bisection for the angle at the anchor when the chain is short and checking both sides of the boundary, obtaining the angle for a weightless line by two routes and matching them**, and **placing the separator on submerged weight**.

1. Notation and the Question

Setting: **in water of depth d a vessel pulls its anchor chain with a horizontal load H.Writing w for the chain's submerged weight per unit length, set**

$$a = \frac{H}{w}$$

a length.The chain hangs under its own weight in a catenary (Paper 327).

Observation 1.1

an anchor works best pulled horizontally.Any upward component pulls it out of the seabed——so the aim is for the chain to meet the seabed at the anchor.

Observation 1.2

the rule is stated as a multiple of the depth.“Pay out five to seven times the depth” is commonly heard——assuming the ratio is constant.

Question: **how long must the chain be for the anchor to be pulled horizontally?Is that proportional to the depth?**

2. The Length Needed, and What Sets the Ratio

Computation 2.1: **with the chain meeting the seabed at the anchor (zero slope there), the catenary relation between arc and height gives the shortest length**

$$L = \sqrt{d^2 + 2da}$$

so that the scope is $L/d = \sqrt{1 + 2a/d}$.

a/d	scope L/d
1	1.732051
5	3.316625
12	5.000000
24	7.000000
50	10.049876

Observation 2.2

this is the core.The scope depends on a/d **alone——a depth of** 10 **m with** $a = 100$ **m and a depth of** 5 **m with** $a = 50$ **m give the same scope to** 12 **digits. Doubling depth and load together leaves the ratio unchanged.**

Observation 2.3

“five to seven” means something.Exactly 5.000000 at $a/d = 12$ and 7.000000 at 24, being $\sqrt{25}$ and $\sqrt{49}$——the rule holds when the horizontal load equals the submerged weight of twelve to twenty-four depths of chain. The condition under which the rule is right can be read from the formula.

Observation 2.4

the inversion was checked.Feeding the length into the catenary relation $d = \sqrt{a^2 + L^2} - a$ returns the depth within 10^{-9}——a comparison with a separately assembled form.

3. Moving the Depth and the Load

Computation 3.1: **load fixed at $a = 50$ m, depth varied.**

depth	length needed	scope
5 m	22.9129 m	**4.582576**
10 m	33.1662 m	3.316625
20 m	48.9898 m	2.449490
40 m	74.8331 m	1.870829
80 m	120.0000 m	**1.500000**

Observation 3.2

deeper means a smaller ratio.At 80 m exactly 1.500000 suffices ($\sqrt{2.25}$)——yet the length itself grows with depth. A falling ratio and a shrinking length are different things.

> **Observation 3.3**
>
> a fixed multiple errs in opposite directions.Holding to “five” at this load falls short in shallow water (5 m needs 4.58) and pays out too much in deep water——with the ratio not constant, no single multiple is right at every depth.

Computation 3.4: depth fixed at 10 m, load varied.

a	length needed	scope
25 m	24.4949 m	2.449490
50 m	33.1662 m	3.316625
100 m	45.8258 m	4.582576
200 m	64.0312 m	6.403124

> **Observation 3.5**
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> doubling the load does not double the chain.From $a = 50$ to 100 the length grows **1.381699**-fold (above 1.3 and below 1.4)——a rising wind does not call for twice the chain, though the same length no longer suffices.

4. A Short Chain Lifts the Anchor

Computation 4.1: at a depth of 10 m and $a = 50$ m (length needed 33.166248 m), shortening the chain and finding the angle at the anchor by bisecting for the position on the catenary.

length relative to that needed	angle at the anchor
1.00	0.000000 degrees
0.95	1.960175 degrees
0.90	4.053067 degrees
0.80	8.736431 degrees
0.70	14.331042 degrees
0.50	30.823826 degrees
0.35	57.722845 degrees

> **Observation 4.2**
>
> both sides of the boundary.Zero at exactly the length needed, positive one per cent below——confirming from both sides that this is the boundary.

> **Observation 4.3**
>
> the angle climbs quickly.About two degrees at five per cent short, over thirty at half (above 30 and below 31) ——how much less the anchor then withstands is outside this paper, but that its pull turns away from the seabed follows from geometry alone.

5. The Separator

Computation 5.1: the same question for a line without submerged weight, which runs straight, meeting the anchor at $\arcsin(d/L)$.

scope	angle at the anchor
5	**11.536959** degrees
7	**8.213211** degrees
100	**0.572967** degrees

Observation 5.2

the separator is whether the line has submerged weight.At the same scope of five, a chain at $a/d = 12$ meets the anchor at exactly zero while a weightless line meets it at 11.536959 degrees——even at a scope of a hundred the weightless line stands at 0.572967 degrees, never horizontal at any finite length. What pulls the anchor flat is weight, not length.

Observation 5.3

the angle agrees by two routes.$\arctan$ from the horizontal distance and $\arcsin(d/L)$ match within 10^{-9}——a comparison between separately assembled forms.

Observation 5.4

a different question from Paper 327.There a hanging chain was told apart from a parabola, the difference lying in whether the load is counted along the arc or the span——here that load counted along the arc, the chain's own weight, is what pulls the anchor horizontally. That paper separates shapes; this one measures a length.

Observation 5.5

the shape of Paper 556.There the members rather than the bolt carried most of the load——here the chain rather than the anchor keeps the pull horizontal: the inconspicuous part doing the work.

Observation 5.6

one root, under Paper 300.“The ratio depends on a/d” and “a short chain lifts the anchor” both come from the one catenary——set the chain’s weight to zero and both give way to another form, the straight line.

6. Roll Call and Closing

1. **length needed $\sqrt{d^2 + 2da}$** (Computation 2.1): **from the catenary.**
2. **scope set by a/d alone** (Observation 2.2): **the core, to 12 digits.**
3. **5.000000 at 12, 7.000000 at 24** (Observation 2.3): **the rule’s condition.**
4. **1.500000 at 80 m** (Observation 3.2): **deeper, smaller ratio.**
5. **4.582576 at 5 m** (Computation 3.1): **shallow water needs nearly five.**
6. **doubling the load, 1.381699** (Observation 3.5): **not double.**
7. **1.960175 degrees at 0.95** (Computation 4.1): **just inside the boundary.**
8. **30.823826 degrees at half** (Computation 4.1): **climbing fast.**
9. **a weightless line at scope five, 11.536959 degrees** (Computation 5.1): **never flat.**
10. **the separator is submerged weight** (Observation 5.2): **weight, not length.**

Closing: **When anchoring, chain is paid out as a multiple of the depth.Yet the ratio of chain to depth is not constant——the shortest chain meeting the seabed at the anchor is $\sqrt{d^2 + 2da}$, with a the load over the chain’s submerged weight, so the scope $\sqrt{1 + 2a/d}$ depends on a/d alone. Doubling depth and load together leaves it unchanged, confirmed to 12 digits by two matched pairs, and the length fed back into the catenary relation returns the depth. The familiar “five to seven times” falls exactly at $a/d = 12$ and 24, returning 5.000000 and 7.000000: the rule is right when the horizontal load equals the submerged weight of twelve to twenty-four depths of chain.At a fixed load, the ratio falls with depth——with $a = 50$ m a depth of 5 m needs 4.582576, while 80 m needs exactly 1.500000; the length itself grows, but the ratio falls. Holding to one multiple therefore falls short in shallow water and pays out too much in deep, and no multiple is right at every depth. Doubling the load lengthens the chain only 1.381699-fold.A chain too short lifts the anchor——1.960175 degrees at 0.95 of the length needed and 30.823826 at half, climbing fast, with zero exactly at the boundary and a positive angle one per cent below. How much less the anchor then holds is outside this paper, but that its pull turns from the seabed follows from geometry alone.The separator is whether the line has submerged weight——at the same scope of five, a chain at $a/d = 12$ meets the anchor at exactly zero while a weightless line meets it at 11.536959 degrees, and even at a scope of a hundred stands at 0.572967, never flat at any finite length; the angle agrees by $\arctan$ and $\arcsin$. What pulls the anchor flat is weight, not length. Under Paper 300 both facts here share one root in the catenary.Placed among the earlier papers——Paper 327**

showed a hanging chain to be a catenary because its load is counted along the arc; this paper uses that very load, the chain's weight, to pull the anchor horizontally, measuring a length where that paper separated shapes. It has the shape of Paper 556 as well, the chain rather than the anchor doing the work.To be honest——no anchoring practice is recommended. Anchor holding capacity, an empirical matter of seabed and anchor form, was excluded, and only the geometry of the angle was examined. The chain was taken as inextensible and uniform on a flat seabed, without dynamic tension or the height of the bow; the load was given as a rather than computed from wind or current. Nor is the rule of thumb rejected: only the range of a/d in which it holds is shown.

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