

A Kaluza–Klein Re-entry

— 2nd ed.: KK Charge Seen Through the Yamagishi Patch — What Looked Like a Wall Was the Same 16π
Counted Twice — [Paper 57]

Yuuki YAMAGISHI (山岸 勇輝)

Yamagishi Complexity Torus Lab, Japan

August 2026 (Second Edition)

— *From thinking to syncing.* —

ABSTRACT

As a sequel to Paper 56, this paper treated the staircase $\Lambda(d) = (4\pi)^{d/2}$, charge as a compact angular momentum, and the elementary charge $e_0 = \sqrt{16\pi G}/R$ given by Kaluza--Klein reduction, presenting the final coefficient as **an explicit open problem, a wall. The second edition shows that the wall was not a wall.** The first edition found it suggestive that the factor **2** inside $\sqrt{16\pi G}$ and the factor **2** of Einstein's equation should be **reached by two independent routes. They are not independent.** The condition $\kappa^2 = 16\pi G_4$ is precisely the instruction to **cancel the prefactor $1/(16\pi G)$ of the five-dimensional Einstein--Hilbert action** in order to canonically normalise the Maxwell term; the **16π is simply the one that was put there at the start, carried through. Two routes did not converge on the same 2: one route produced it once** (Observation 4.3). This is the shape Paper 51 exhibited for the shell theorem — **count the same object twice and the agreement looks like a coincidence. Second, the staircase carries the Fourier convention at every rung.** The third edition of Paper 46 showed that at $d = 4$ the loop factor is **a Jacobian times a convention divided by a geometry**; this holds in general, since $(4\pi)^{d/2}\Gamma(d/2) = 2(2\pi)^d/\Omega_{d-1}$ for every d (verified symbolically here). **The even/odd observation nevertheless survives**, because the $\pi^{d/2}$ comes from $\Omega_{d-1} = 2\pi^{d/2}/\Gamma(d/2)$ — **that half is geometry** — while what carries the convention is the 2^d . **The finding stands; its reason changes** (Observation 2.3). **Third, charge cannot be an angular momentum.** The mass dimension of mvr is **1** and that of Gaussian charge is **$1/2$** (Paper 56, 2nd ed.; Paper 116). **The equality does not hold.** What remains is Kaluza--Klein's own statement — that p_5 is quantised by single-valuedness and $p_5 R \in \mathbb{Z}$ — and **that needs no patch at all** (Observation 3.3). **Fourth, “the concentration of critical dimensions” said too much.** The first pole of $\Gamma(n - d/2)$ is at $d = 2n$, and $d = 4$ is the case $n = 2$. Moreover the location of the pole is a fact about **the analytic structure of Γ** while collapse to the centre is a fact about **the boundedness of a Schrödinger operator**: distinct roots. This is **brought back to the strength Paper 56 used of itself** — “selection by consistency, not derivation” (Observation 2.5).

Scope of this paper: NO NEW PHYSICAL LAW IS CLAIMED. All external physics is standard (Kaluza 1921, Klein 1926, dimensional regularisation, Ehrenfest 1917). Every correction added in the second edition comes from later papers of this corpus (46, 51, 56 2nd ed., 116).

1. Notation, assumptions, position

As established in Paper 56, the literal reading “the power of $\mathfrak{c}$ is the number of dimensions” is not supported, since the exponent is fixed uniquely by dimensional analysis; and integration with respect to $\mathfrak{c}$ is invalid because $\mathfrak{c}$ is a constant (withdrawn in Paper 34). This paper therefore treats **the dimension d , not $\mathfrak{c}$, as the continuous variable** — the same ground as dimensional regularisation, the heat kernel and ζ regularisation.

Position: This paper **claims no new physical law**. It places the corpus's quantities within the known structure of Kaluza--Klein theory and **separates what can be settled from what cannot**.

Added in the second edition: The first edition left one point standing as **a wall**. **The second edition inspects that wall and reports that it was not one.**

2. The staircase $\Lambda(d) = (4\pi)^{d/2}$

Proposition 2.1 (closed form of the staircase)

The loop and measure factor is

$$\Lambda(d) = (4\pi)^{d/2} = 2^d \pi^{d/2}$$

an integer power of π in even dimensions and a half-integer power in odd ones (verified here):

d	$\Lambda(d) = 2^d \pi^{d/2}$	Power of π
2	4π	integer
3	$8\pi^{3/2}$	half-integer
4	$16\pi^2$	integer (our spacetime)
5	$32\pi^{5/2}$	half-integer
6	$64\pi^3$	integer

Corollary 2.2 (the first edition's negative result, unchanged)

The 4π , $16\pi^2$ and $64\pi^3$ of the corpus are **the even-dimensional rungs**. Five dimensions gives $32\pi^{5/2}$, a **half-integer power of π** , and does not sit on the integer staircase. **There is no “clean fifth dimension” on this reading.**

2.1 [Second edition] Every rung carries the Fourier convention

Observation 2.3

The third edition of Paper 46 showed that at $d = 4$ the loop factor's $16\pi^2$ is a **Jacobian times the Fourier convention divided by a geometry**, and that changing the convention changes the number. **This is not confined to $d = 4$.** For every d ,

$$(4\pi)^{d/2} \Gamma(d/2) = \frac{2(2\pi)^d}{\Omega_{d-1}}, \quad \Omega_{d-1} = \frac{2\pi^{d/2}}{\Gamma(d/2)} \quad (2)$$

(verified symbolically here for all d), where the **2** is the Jacobian of the change of variable, the $(2\pi)^d$ is the **convention** of where the **2 π** of the Fourier transform is placed, and Ω_{d-1} is the area of the unit $(d-1)$ -sphere, that is the **geometry**. **Every rung of the staircase contains all three.**

Observation 2.4 (the even/odd observation survives all the same)

On the right of (2), what produces the power of π is $\Omega_{d-1} = 2\pi^{d/2}/\Gamma(d/2)$, that is **the sphere**. In terms of the split $\Lambda(d) = 2^d \pi^{d/2}$:

Factor	Where it comes from	Standing
2^d	the 2^d of the convention $(2\pi)^d$	convention
$\pi^{d/2}$	the sphere's area Ω_{d-1}	geometry

The statement that even dimensions give integer powers and odd ones half-integer powers is a statement about $\pi^{d/2}$, and therefore **lives on the geometric side**. The negative result of Corollary 2.2 — that five dimensions does not sit on the integer staircase — **holds independently of the choice of convention**.

The finding stands, and its reason has changed. Calling it “the 4π staircase” is not accurate: it is **the sphere's $\pi^{d/2}$ multiplied by the convention's 2^d** .

2.2 [Second edition] “The concentration of critical dimensions” said too much

The first edition noted that (i) the first pole of the loop integral $\Gamma(2 - d/2)$ is at $d = 4$, and (ii) the threshold for collapse to the centre in an attractive potential is spatial $d = 4$, and said that the critical dimensions **concentrate** at $d = 4$.

Observation 2.5 (two corrections)

First, (i) is a statement about a particular integral. For a propagator power n the pole sits in $\Gamma(n - d/2)$ and the first one is at $d = 2n$ ($d = 2$ for $n = 1$, $d = 4$ for $n = 2$, $d = 6$ for $n = 3$). “The first pole is at $d = 4$ ” is the case $n = 2$.

Second, (i) and (ii) have distinct roots. The first is a fact about **the analytic structure of the Γ function**; the second about **whether a Schrödinger operator is bounded below**. That they point to the same d is a fact, **but not for the same reason**.

The word “concentration” is therefore **withdrawn**. Paper 56 limited a comparable situation correctly as “**a selection by consistency, not a derivation**”, and the second edition returns to that strength: $d = 4$ is the dimension at which several independent requirements **happen to return the same answer**.

3. Charge as a compact angular momentum, and single-valuedness as synchronisation

In Kaluza–Klein theory spacetime is four dimensions plus a compact circle S^1 of radius R (with $y \sim y + 2\pi R$), and fields expand as $\psi(x, y) = \sum_n \psi_n(x) e^{iny/R}$.

Proposition 3.1 (charge quantisation from single-valuedness)

For a field to be single-valued around the circle,

$$p_5 \cdot (2\pi R) = 2\pi n \implies p_5 = \frac{n}{R} \quad (n \in \mathbb{Z})$$

and this p_5 appears in four dimensions as electric charge. **Charge quantisation follows from the phase coming back into step after one turn around the circle.**

Corollary 3.2 ($p_5 R$ as an angular momentum)

$p_5 R = n \in \mathbb{Z}$, so the angular momentum around the circle takes integer values. **This statement closes entirely inside Kaluza--Klein theory and requires no patch whatever.**

3.1 [Second edition] $e = 4\pi v r$ cannot be identified with an angular momentum

The first edition stated that under the mass patch $m = 4\pi$ the relation $e = 4\pi v r = m v r$ coincides with the angular momentum $L = p_5 R$ around the circle.

Observation 3.3 (the mass dimensions do not match)

$[m v r] = M \cdot (L/T) \cdot L = M L^2 T^{-1}$, that is **mass dimension $\mu = 1$** ; Gaussian charge has $[e] = M^{1/2} L^{3/2} T^{-1}$, that is $\mu = 1/2$.

Since $1 \neq 1/2$, $e = m v r$ does not hold as an equality of quantities. As Paper 116 showed, the mass dimension is precisely the power of the patch constant ($\varepsilon = \mu$) and **does not move when the representation changes**. The second edition of Paper 56 withdrew, on this ground, the general claim that charge is a geometric quantity. **The claim that charge is an angular momentum is withdrawn here for the same reason.**

Observation 3.4 (what remains)

What remains is **Kaluza--Klein's own statement**: single-valuedness quantises p_5 and $p_5 R$ is an integer. This holds **with no patch at all**, and **the picture of synchronisation — a phase coming back into step after one turn — already lives there.**

The structure the first edition saw is real. What is not real is joining it by an equals sign to the corpus's $e = 4\pi v r$. The synchronisation picture was already inside Kaluza--Klein, and did not need to pass through the patch.

4. The $16\pi G$ coefficient, and what the first edition called a wall

Proposition 4.1 (the elementary charge from KK reduction)

Reducing the five-dimensional Einstein--Hilbert action $S_5 = \frac{1}{16\pi G_5} \int d^5x \sqrt{-g_5} R_5$ with the ansatz $ds_5^2 = g_{\mu\nu} dx^\mu dx^\nu + (dy + \kappa A_\mu dx^\mu)^2$ and canonically normalising the Maxwell term to $-\frac{1}{4}F^2$ gives $\kappa = \sqrt{16\pi G_4}$, so that the n -th mode carries

$$q_n = \kappa \frac{n}{R} = \frac{\sqrt{16\pi G}}{R} n, \quad e_0 = \frac{\sqrt{16\pi G}}{R}$$

and at $G = 4\pi$, $e_0 = \sqrt{16\pi \cdot 4\pi}/R = \sqrt{64\pi^2}/R = 8\pi/R$ (verified here).

Corollary 4.2 (4π and 8π are different quantities, unchanged)

The 4π comes from the mass patch and the $8\pi = \sqrt{16\pi G}|_{G=4\pi}$ from the normalisation of the coupling. **They play different roles and there is no contradiction. The second edition adds one remark:** the patch constant may be any positive number (Paper 116), so this is a contrast of **convention with convention**.

4.1 [Second edition] The wall was not a wall

The first edition wrote that the factor **2** inside $\sqrt{16\pi G}$ is the same as the factor **2** of Einstein's equation, and that **two independent routes** — the gravitational action and the charge coupling — **land on exactly the same factor**, which it called suggestive.

Observation 4.3 (the two routes are one route)

They are not independent. One need only follow where the condition $\kappa^2 = 16\pi G_4$ comes from.

After reduction the action takes the form

$$S = \frac{1}{16\pi G_4} \int d^4x \sqrt{-g} \left[R_4 - \frac{\kappa^2}{4} F^2 \right]$$

and bringing the Maxwell term to the canonical $-\frac{1}{4}F^2$ requires

$$\frac{1}{16\pi G_4} \cdot \frac{\kappa^2}{4} = \frac{1}{4} \implies \kappa^2 = 16\pi G_4$$

That is, $\kappa^2 = 16\pi G_4$ is nothing but the instruction “cancel the prefactor $1/(16\pi G)$ you started with”.

The 16π appearing in κ was therefore not reached independently. It is the prefactor of the five-dimensional Einstein–Hilbert action, carried through the reduction and then cancelled. The same 16π appears in two places: where it was put, and where it is undone.

What the first edition called “two independent routes converging on the same 2” is one route producing that 2 once. There is no convergence. The judgement that it is suggestive is withdrawn.

Observation 4.4 (a shape this corpus keeps meeting)

Paper 51 showed that in the shell theorem the 4π appears on both sides — the area of the sphere and the normalisation of Gauss's law — as the same solid angle, and cancels. When the same object appears twice, the agreement looks like a coincidence. The 16π here is the same: what was put in came back.

The question that remains, restated correctly: why does the prefactor of the Einstein–Hilbert action contain the 2 of $16\pi G = 2 \cdot 8\pi G$? That comes from trace reversal in the Newtonian limit (known). And the question Paper 01 left open — whether that 2 shares an origin with the “two sheets” of the mass ladder — receives no new evidence from the KK reduction of this paper, since KK merely carried the 16π and says nothing about its contents. The wall was not solved; it was not this paper's wall.

5. Conclusion

1. The staircase is $\Lambda(d) = (4\pi)^{d/2} = 2^d \pi^{d/2}$, an integer power of π in even dimensions and a half-integer power in odd ones (Proposition 2.1). Five dimensions gives $32\pi^{5/2}$ and does not sit on the integer staircase (Corollary 2.2).
2. [2nd ed.] Every rung carries the Fourier convention — $(4\pi)^{d/2} \Gamma(d/2) = 2(2\pi)^d / \Omega_{d-1}$ for all d (verified). The even/odd observation nevertheless survives: the $\pi^{d/2}$ comes from the sphere's area and is geometry, while the convention is carried by the 2^d (Observations 2.3, 2.4). The finding stands and its reason has changed.
3. [2nd ed.] “The concentration of critical dimensions” is withdrawn — the first pole of $\Gamma(n - d/2)$ is at $d = 2n$, and $d = 4$ is the case $n = 2$; and the pole and the collapse have distinct roots (Observation 2.5). The strength of “selection by consistency” used in Paper 56 is restored.
4. In KK, charge appears as a quantised momentum around S^1 and charge quantisation follows from single-valuedness (Proposition 3.1, Corollary 3.2). That statement needs no patch.

5. [2nd ed.] The identification of $e = 4\pi vr$ with an angular momentum is withdrawn — $[mvr]$ has mass dimension 1 and Gaussian charge $1/2$, so the equality fails (Observation 3.3). The synchronisation picture was already inside KK and did not need the patch (Observation 3.4).
6. Full KK reduction gives $e_0 = \sqrt{16\pi G}/R = 8\pi/R$ at $G = 4\pi$ (Proposition 4.1). That 4π and 8π are different quantities is right, but both are conventions (Corollary 4.2).
7. [2nd ed.] What the first edition called a wall was not a wall — $\kappa^2 = 16\pi G_4$ is the instruction to cancel the prefactor one started with, and the 16π is the same 16π carried through. Two independent routes did not converge on the same 2; one route produced it once (Observation 4.3), the shape Paper 51 exhibited for the shell theorem (Observation 4.4). This paper supplies no new evidence on Paper 01's open question.

Closing: The first edition left a wall standing at the end. What the second edition did was knock on it. The wall was not solved — it was never a wall. What had been put in came back, and the agreement looked like one only because the same number was being counted twice. Finding that what one took for an open problem was never a question is also a step forward.

References

1. Kaluza, Th. (1921). *Zum Unitätsproblem der Physik*. Sitzungsberichte der Preussischen Akademie der Wissenschaften Berlin, 966--972.
2. Klein, O. (1926). *Quantentheorie und fünfdimensionale Relativitätstheorie*. Zeitschrift für Physik, 37, 895--906.
3. Ehrenfest, P. (1917). *In what way does it become manifest in the fundamental laws of physics that space has three dimensions?* Proceedings of the Amsterdam Academy, 20, 200--209.
4. Wigner, E. P. (1939). *On unitary representations of the inhomogeneous Lorentz group*. Annals of Mathematics, 40, 149--204.
5. 't Hooft, G., & Veltman, M. (1972). *Regularization and renormalization of gauge fields*. Nuclear Physics B, 44, 189--213.
6. Yamagishi, Y. (2026). *The Uniqueness Theorem of $\mathfrak{n} = 2$ for Spherically Symmetric Density Distributions* [Paper 01]. Zenodo. 10.5281/zenodo.21403186
7. Yamagishi, Y. (2026). *A Convention Is a Number, Not a Function of the Configuration* [Paper 15], 3rd ed. Zenodo. 10.5281/zenodo.21600352
8. Yamagishi, Y. (2026). *A Census of the $\mathfrak{c}^5$ s* [Paper 34], 3rd ed. Zenodo. 10.5281/zenodo.21430340
9. Yamagishi, Y. (2026). *The Power of 4π Is the Mass Dimension* [Paper 44], 3rd ed. Zenodo. 10.5281/zenodo.21602231
10. Yamagishi, Y. (2026). *Of $16\pi^2$, Only $\Omega_3 = 2\pi^2$ Is Geometry* [Paper 46], 3rd ed. Zenodo. 10.5281/zenodo.21602422
11. Yamagishi, Y. (2026). *The Yamagishi Gravitational Multipole Aggregation Theory* [Paper 51], 3rd ed. Zenodo. 10.5281/zenodo.21602935
12. Yamagishi, Y. (2026). *A Structural Study of the 4π Patch* [Paper 56], 2nd ed. Zenodo. 10.5281/zenodo.21597500

13. Yamagishi, Y. (2026). *Convention and Fact* [Paper 95]. Zenodo. 10.5281/zenodo.21758621
14. Yamagishi, Y. (2026). *What Does the Exponent of $\mathfrak{c}$ Count?* [Paper 116]. Zenodo. 10.5281/zenodo.22003794

— *From thinking to syncing.* —

Yuuki YAMAGISHI (山岸 勇輝)

© 2026 Yamagishi Complexity Torus Lab, Japan · Licensed under Creative Commons Attribution 4.0 International (CC BY 4.0)