Between Unlike Materials, an Interference Fit Vanishes with Temperature

— A **50** mm fit assembled with **25** μm of interference is untouched by temperature in steel on steel, yet with a steel shaft in an aluminium hub the interference vanishes after a rise of **44.247788** K — and the aluminium hub gives only **0.405521** of the steel hub's contact pressure for the same interference — [Paper 570]

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— *From thinking to syncing.* —

Abstract

An interference fit, once assembled, is treated as mechanically fixed.What this paper shows is that when shaft and hub differ in material the interference becomes a function of temperature, and the direction in which it loosens is set by which material is outside.No new theorem or law is claimed. First, the contact pressure follows from thick-cylinder theory——38.6250 MPa in steel on steel and 15.6632 MPa with an aluminium hub, a ratio of 0.405521 (Section 2). **Second, the formula returns to a known form in the limit——one material with an infinitely thick hub gives $E\delta/(2D) = 51.500000$ MPa** (Section 2). **Third, and this is the core.A steel shaft in an aluminium hub loses its interference after a rise of 44.247788 K——in one material the change is exactly zero at every temperature** (Section 3). **Fourth, the pressure falls in a straight line——half the rise gives exactly half the pressure, to 12 digits** (Section 3). **Fifth, reversing the pair reverses the direction——an aluminium shaft in a steel hub gains exactly 25 μm over the same rise** (Section 4). **Sixth, the separator is whether shaft and hub expand alike——if they do, temperature has no effect; if not, the sign of the difference sets the direction of loosening** (Section 5). **Keywords**: interference fit, interference, thermal expansion, thick-walled cylinder, contact pressure.

Scope of this paper (scope note): **No new theorem or law is claimed——thick-cylinder contact pressure and thermal expansion are standard.No fit is designed——which tolerance grade to choose is not stated.Only the elastic range is treated——neither yield nor creep is included, though creep matters in a hot aluminium hub.Temperature is uniform ——shaft and hub are taken at one temperature; differences in how fast they warm are not examined.Material constants are representative——expansion coefficients 11.8×10^{-6} and 23.1×10^{-6} per kelvin, moduli 206 and 70 GPa, Poisson's ratios 0.30 and 0.33.Holding capacity is not computed——with no friction coefficient, the force or torque the fit transmits is not stated.Loosening by rotation is excluded——the hub's centrifugal expansion is not included.Unlike materials are not discouraged——knowing the temperature range, the interference can be chosen to survive it.Relation to earlier papers**: **Paper 562 showed that what caps a residual stress is not the temperature but the yield——there restrained thermal contraction raised a stress capped by yield; here the difference in expansion between two materials eats the interference. The same $\alpha \Delta T$ raises a stress in one and undoes an assembly in the other.Paper 548 showed that the same z works in opposite directions according to whether it multiplies a denominator or a numerator——the same shape here: one rise in temperature loosens or tightens according to which material is outside.Paper 300 showed that sharing a root is decidable——"there is a temperature at which the interference vanishes" and "reversing the pair reverses the direction" share one root in the difference of expansion.**What is added is **computing the contact pressure by thick-cylinder theory for unlike materials and confirming its return to the known $E\delta/(2D)$ in the limit**, **finding the loss of interference at 44.247788 K and checking both sides of it**, **confirming the straight-line fall of pressure at the halfway point to 12 digits**, **showing the mirror of the reversed pair and the zero of the like pair**, and **placing the separator on equal expansion**.

1. Notation and the Question

Setting: **a solid shaft of diameter D is pressed into the bore of a hub of outer diameter D_o, made larger by a diametral interference δ.Thick-cylinder theory gives the contact pressure**

$$p = \frac{\delta}{D\left[\frac{1}{E_o}\left(\frac{D_o^2 + D^2}{D_o^2 - D^2} + \nu_o\right) + \frac{1 - \nu_i}{E_i}\right]}$$

with subscript i for the shaft and o for the hub.

Observation 1.1

interference is a difference of dimensions.Dimensions change with temperature——if shaft and bore grow alike the difference holds, and if they grow differently it moves.

Question: **what happens to interference and pressure as the temperature rises?And how does that depend on the pair of materials?**

2. Contact Pressure

Computation 2.1: **with $D = 50$ mm, $D_o = 100$ mm and $\delta = 25\ \mu$m.**

pair	contact pressure
steel shaft, steel hub	**38.6250** MPa
steel shaft, aluminium hub	**15.6632** MPa
ratio	**0.405521**

> **Observation 2.2**
>
> **an aluminium hub gives four tenths.Above 0.40 and below 0.41——with a modulus about a third of steel's, the same difference in size pushes back more weakly.**

> **Observation 2.3**
>
> **the formula returns to a known form.One material and an infinitely thick hub give 51.500000 MPa, matching the closed form $E\delta/(2D)$ to a relative 10^{-6}——a comparison with a separately assembled formula.**

3. Temperature Takes the Interference Away

Computation 3.1: **a steel shaft (11.8×10^{-6} per kelvin) in an aluminium hub (23.1×10^{-6}), temperature raised.The bore widens faster, so the interference falls by $(\alpha_o - \alpha_i) D \Delta T$.**

rise	interference	contact pressure
0 K	25.0000 μm	**15.6632** MPa
10 K	19.3500 μm	12.1234 MPa
20 K	13.7000 μm	8.5835 MPa
30 K	8.0500 μm	5.0436 MPa
40 K	2.4000 μm	1.5037 MPa
44.247788 K	0	0

Observation 3.2

this is the core.The interference vanishes after a rise of **44.247788** K (above 44 and below 45)——a difference like that between summer and winter, or running and standing, undoes a joint meant to be fixed.

Observation 3.3

both sides of the boundary were checked.At that temperature the pressure is zero, under 10^{-9} of its initial value, and one per cent before it is still positive——the first version of the check compared a floating-point result with exactly zero and failed; the fault was in how the check was written, and it now carries a tolerance.

Observation 3.4

the pressure falls in a straight line.Half the rise gives exactly half the pressure, to 12 digits——the pressure is proportional to the interference, which falls linearly with temperature. Nothing collapses suddenly; the grip is shaved evenly from the start.

4. Reversing the Pair Reverses the Direction

Computation 4.1: the same two materials with the shaft of aluminium and the hub of steel.Now the shaft swells faster, and the same rise of **44.247788** K adds exactly 25 μm.

Observation 4.2

a mirror image.The gain equals the original interference to 12 digits——at the temperature where one pair comes loose, the other doubles its interference.

Observation 4.3

the reversed pair loosens on cooling.An aluminium shaft in a steel hub loses its interference after cooling by the same 44.247788 K——each pair has a direction of temperature in which it loosens; they differ only in whether it lies up or down.

Computation 4.4: **in one material, steel shaft in steel hub, the change in interference is exactly zero at every temperature.**

Observation 4.5

in one material temperature has no effect at all.Shaft and bore grow in the same proportion and the difference stays put——“mechanically fixed” is accurate only here.

5. The Separator

Observation 5.1

the separator is whether shaft and hub expand alike.If they do, temperature has no effect——if not, the sign of $\alpha_o - \alpha_i$ sets the direction of loosening. An outer part that expands more loosens when warmed; an inner part that expands more loosens when cooled.

Observation 5.2

the shape of Paper 548.There the same z worked in opposite directions as denominator or numerator——here one rise in temperature loosens or tightens according to which material is outside. One quantity, and its placement sets the direction.

Observation 5.3

a different use of $\alpha \Delta T$ from Paper 562.There restrained thermal contraction raised a stress capped by yield ——here a difference of expansion undoes the assembly itself. The same thermal expansion is a source of stress in one and a dissolver of joints in the other.

Observation 5.4

one root, under Paper 300.“There is a temperature at which the interference vanishes” and “reversing the pair reverses the direction” both come from the difference of expansion——make it zero and both vanish.

Observation 5.5

what is not said.Unlike materials are not discouraged——knowing the range of temperature in use, the interference can be chosen to survive it. What is shown is what happens when a fit is made without knowing that range.

6. Roll Call and Closing

1. **steel on steel, $\mathbf{38.6250}$ MPa** (Computation 2.1): **thick-cylinder theory.**
2. **aluminium hub, $\mathbf{15.6632}$ MPa, ratio $\mathbf{0.405521}$** (Computation 2.1): **four tenths.**
3. **the limit $E\delta/(2D) = \mathbf{51.500000}$ MPa** (Observation 2.3): **a known form.**
4. **interference lost at $\mathbf{44.247788}$ K** (Computation 3.1): **the core.**
5. **zero there, positive just before** (Observation 3.3): **both sides.**
6. **half the rise, half the pressure** (Observation 3.4): **a straight line, to 12 digits.**
7. **the reversed pair gains exactly 25 μm** (Computation 4.1): **a mirror.**
8. **one material, exactly zero change** (Computation 4.4): **temperature has no effect.**
9. **the separator is equal expansion** (Observation 5.1): **the sign sets the direction.**
10. **the shape of Paper 548** (Observation 5.2): **placement sets the direction.**

Closing: **An interference fit, once assembled, is treated as mechanically fixed.Yet interference is a difference of dimensions, and dimensions change with temperature——at 50 mm and 25 μm, steel on steel gives $\mathbf{38.6250}$ MPa and an aluminium hub $\mathbf{15.6632}$ MPa, a ratio of $\mathbf{0.405521}$, and the formula returns in the limit of one material and**

an infinitely thick hub to the known $E\delta/(2D) = 51.500000$ MPa.A steel shaft in an aluminium hub loses its interference after a rise of 44.247788 K——the bore widens faster, the interference is shaved linearly, and the pressure falls in a straight line, exactly half at half the rise. At that temperature the pressure is zero and one per cent before it still positive, confirmed from both sides. A difference like summer and winter, or running and standing, undoes a joint meant to be fixed.Reversing the pair reverses the direction——an aluminium shaft in a steel hub gains exactly 25 μm over the same rise, doubling its interference, and instead loses it on cooling by the same amount. Each pair has a direction in which it loosens, up or down. In one material the change is exactly zero at every temperature, and "mechanically fixed" is accurate only there.The separator is whether shaft and hub expand alike ——if they do, temperature has no effect; if not, the sign of $\alpha_o - \alpha_i$ sets the direction, an outer part that expands more loosening when warmed and an inner one loosening when cooled. Under Paper 300 the vanishing temperature and the reversal share one root in the difference of expansion.Placed among the earlier papers——it has the shape of Paper 548, where the same z worked oppositely as denominator or numerator, here one rise working oppositely according to which material is outside. It uses $\alpha\Delta T$ differently from Paper 562, as a dissolver of joints rather than a source of stress.To be honest——no fit is designed. Only the elastic range was treated, without the yield or the creep that matters in a hot aluminium hub; shaft and hub were held at one temperature; the constants are representative; with no friction coefficient no holding capacity is stated; and rotation was excluded. Nor are unlike materials discouraged: knowing the range of temperature, the interference can be chosen to survive it.

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