Lower the Water Table by 1 m and the Bottom of the Fresh Water Rises 40 m

— Sea water is only **2.5** per cent denser than fresh water, and because that difference sits in the denominator, fresh water reaches down **40.000000** times the height of the water table above sea level — a change in sea water from **1025** to **1030** alone drops the ratio to **33.333333** — [Paper 574]

Yuuki YAMAGISHI (山岸 勇輝)
Yamagishi Complexity Torus Lab, Japan
September 2026

— From thinking to syncing. —

Abstract

Under islands and coasts, light fresh water floats on heavy sea water.What this paper shows is that the depth of the bottom of the fresh water is $\rho_f/(\rho_s-\rho_f)$ times the height of the water table above sea level, and because the denominator is a difference of densities, the ratio is an amplifier that turns a small difference into a great depth.No new theorem or law is claimed. First, the ratio is set by the difference of densities——with fresh water at 1000 and sea water at 1025 kg/m^3, $\rho_f/(\rho_s-\rho_f)=$ 40.000000 (Section 2). **Second, the ratio is sensitive to the difference——sea water at 1020 gives 50.000000 and at 1030 gives 33.333333; a change of 0.5 per cent in density changes the ratio by a factor of 0.8333** (Section 2). **Third, and this is the core.Lower the water table and the bottom of the fresh water rises forty times as far——2.0000 m for 0.05 m, 4.0000 m for 0.10 m and 10.0000 m for 0.25 m** (Section 3). **Fourth, the fresh-water lens of a strip island can be written down——with a half-width of 500 m, recharge of 300 mm a year and conductivity of 20 m a day, the water table stands at most 0.500584 m above sea level, the bottom lies at 20.0234 m and the lens is 20.5240 m thick** (Section 4). **Fifth, pumping 0.25 m of that down brings the bottom up to 10.0234 m——halving the water table above sea level nearly halves the fresh water** (Section 3). **Sixth, doubling the difference of densities does not halve the lens——taking the ratio from 40 to 20 changes the thickness by 0.715678, matching the closed form $\sqrt{21/41}$** (Section 4). **Keywords**: saltwater intrusion, Ghyben-Herzberg relation, groundwater, freshwater lens, pumping.

Scope of this paper (scope note): No new theorem or law is claimed——the Ghyben--Herzberg relation and the Dupuit approximation are standard.A sharp interface in still water is assumed——in reality a mixed zone lies between fresh and sea water and moves with the tide; neither is included.The time to rise is not treated——the fortyfold rise after pumping holds once equilibrium is reached.The Dupuit approximation is used——flow is taken as nearly horizontal and vertical flow is dropped.The numbers are representative——fresh water 1000 and sea water 1025 kg/m^3, a half-width of 500 m, recharge of 300 mm a year and conductivity of 20 m a day.The island is a strip——a round island changes the coefficients.No well is designed——where and how much to pump is not stated.Relation to earlier papers: Paper 452 showed that what lifts a liquid is not surface tension but its difference——here too a difference is the protagonist, but there it drives a flow and here it sets a depth, and sitting in the denominator it becomes an amplifier.Paper 408 stated in its scope note that groundwater was not its subject——this paper takes that seat: there salt accumulates in the land from above, here sea water rises from below.Paper 300 showed that sharing a root is decidable——"forty times the height of the water table" and "the ratio moves strongly with the density of sea water" come from one root, a difference in the denominator.What is added is showing by varying the density of sea water that the ratio of 40 is a gain born of the smallness of a difference, setting out the rise of the interface when the water table is pumped down, writing the lens of a strip island by the Dupuit formula and checking numerically that it satisfies the flow equation, and checking the thickness ratio 0.715678 for a doubled difference against the closed form $\sqrt{21/41}$.

1. Notation and the Question

Setting: let ρ_f be the density of fresh water, ρ_s that of sea water, h the height of the water table above sea level and z the depth below sea level of the bottom of the fresh water (the interface with sea water).Balancing pressure at the interface, $\rho_f g(h+z) = \rho_s g z$, gives

$$z = \frac{\rho_f}{\rho_s - \rho_f}\, h = \alpha\, h.$$

The fresh water is $(1+\alpha)h$ thick.

Observation 1.1

fresh water is lighter than sea water, so it floats.However little of it shows above sea level, it reaches deep below——the shape of an iceberg, but with a much smaller difference of density.

Question: how deep does the fresh water reach?What happens when the water table is lowered a little?

2. The Ratio Is Set by a Difference

Computation 2.1: **fresh water at 1000 kg/m^3, with the density of sea water varied.**

density of sea water	$\alpha = \rho_f/(\rho_s - \rho_f)$
1020 kg/m^3	**50.000000**
1025 kg/m^3	**40.000000**
1030 kg/m^3	**33.333333**

> **Observation 2.2**
>
> **a difference in the denominator makes the ratio large.Sea water is only 2.5 per cent denser than fresh—— that smallness is exactly what becomes the 40. A quantity with a difference in its denominator is an amplifier that grows as the difference shrinks.**

> **Observation 2.3**
>
> **the ratio is sensitive to the difference.A change of 0.5 per cent in sea water, from 1025 to 1030, changes the ratio by a factor of 0.8333——the density itself hardly moves, but the difference grows 1.2 times, so the ratio shrinks by the same factor.**

3. Pumping Down

Computation 3.1: **in the still-water limit after equilibrium, the rise $\alpha\Delta h$ of the interface when the water table is lowered by Δh.**

lowering of the water table	rise of the bottom of the fresh water
0.05 m	**2.0000** m
0.10 m	**4.0000** m
0.25 m	**10.0000** m

Observation 3.2

this is the core.Lower the water table by 1 and the bottom of the fresh water rises by 40——a well that lowers the water table only slightly draws sea water up a long way from below. The harm arrives amplified fortyfold.

Observation 3.3

on the island of the next section, pumping half the table away nearly halves the fresh water.Lowering the peak water table of 0.500584 m by 0.25 m raises the bottom of the fresh water from 20.0234 m to 10.0234 m——a visible change of 25 cm in the water table takes 10 m of fresh water from underfoot.

4. The Lens of a Strip Island

Computation 4.1: rain of R a year soaks into a strip island of half-width a and flows through a layer of conductivity K to the sea on both sides.In the Dupuit approximation, balancing the flow through a thickness $(1+\alpha)h$ against recharge gives

$$h(x)^2 = \frac{R\,(a^2 - x^2)}{K\,(1+\alpha)}.$$

With $a = 500$ m, $R = 300$ mm a year and $K = 20$ m a day:

quantity	value
peak water table above sea level	0.500584 m
depth of the bottom of the fresh water	20.0234 m
thickness of the lens	20.5240 m

Observation 4.2

the formula was checked numerically against the flow equation.The residual built from second differences of h^2 is under 10^{-6} of the recharge R——a slip in the formula would show a large residual here.

Observation 4.3

50 cm shows above sea level, and forty-one times that lies underfoot.Most of the island's fresh water lies below sea level——the water table above sea level is only a small marker of how much there is.

Computation 4.4: the same island with the difference of densities doubled (the ratio from 40 to 20).

ratio	peak water table	thickness of the lens
40	**0.500584** m	**20.5240** m
20	**0.699455** m	**14.6885** m

Observation 4.5

doubling the difference does not halve the lens.The thickness changes by **0.715678**, matching the closed form $\sqrt{21/41}$ to 10^{-12}——a smaller ratio makes the bottom shallower, but the water table rises to carry the flow; the two partly cancel, and the effect weakens to a square root.

5. The Separator

Observation 5.1

the separator is whether the difference of densities sits in the denominator.A quantity with a difference in its denominator grows as the difference shrinks and is sensitive to its changes——the mere 2.5 per cent between fresh and sea water makes both the ratio of 40 and the fortyfold amplification of pumping.

Observation 5.2

one root, under Paper 300.“Reaching forty times the water table” and “the ratio moves strongly with the density of sea water” come from one root, a difference in the denominator——enlarge the difference and both weaken.

Observation 5.3

as in Paper 452, a difference is the protagonist.There, what lifted the liquid was not surface tension but its difference——there the difference drives a flow, here it sets a depth; and here it sits in the denominator, so it becomes an amplifier.

Observation 5.4

taking the seat reserved by Paper 408.Its scope note set groundwater aside——there salt accumulates in the land from above, here sea water rises from below. Both say that fresh water underground is more fragile than it looks.

Observation 5.5

what is not said.It is not said that wells should not be dug on islands——with the right amounts and places, the fresh water can be used. What is shown is the size of the amplification, by which a slight change in the water table acts forty times over underfoot.

6. Roll Call and Closing

1. **a ratio of 40.000000** (Computation 2.1): **a difference in the denominator.**
2. **33.333333 for sea water at 1030** (Computation 2.1): **0.5 per cent gives a factor of 0.8333.**
3. **0.25 m down, the bottom 10.0000 m up** (Computation 3.1): **the core.**
4. **0.500584 m above sea level, bottom at 20.0234 m** (Computation 4.1): **the lens of a strip island.**
5. **a lens 20.5240 m thick** (Computation 4.1): **forty-one times the visible table.**
6. **residual under 10^{-6} of R** (Observation 4.2): **the formula checked.**
7. **pumping half down brings the bottom to 10.0234 m** (Observation 3.3): **nearly half the fresh water gone.**
8. **doubling the difference gives 0.715678** (Observation 4.5): **the closed form $\sqrt{21/41}$.**
9. **the separator is a difference in the denominator** (Observation 5.1): **the condition for an amplifier.**
10. **the seat reserved by Paper 408** (Observation 5.4): **salt from above, sea water from below.**

Closing: **Under islands and coasts, light fresh water floats on heavy sea water.The bottom of the fresh water lies $\rho_f/(\rho_s-\rho_f)$ times the height of the water table below sea level——with fresh water at 1000 and sea water at 1025**

kg/m^3 the ratio is 40.000000, large because its denominator is a difference of 2.5 per cent. With sea water at 1030 it is 33.333333 and at 1020 it is 50.000000; a change of 0.5 per cent in density changes the ratio by a factor of 0.8333.Lower the water table and the bottom of the fresh water rises forty times as far——10.0000 m for 0.25 m. On a strip island of half-width 500 m a water table of 0.500584 m above sea level holds fresh water down to 20.0234 m, a lens 20.5240 m thick with forty-one times the visible table underfoot, and the formula satisfies the flow equation to a residual under 10^{-6} of R. Pumping 0.25 m of that down brings the bottom up to 10.0234 m. Doubling the difference of densities does not halve the lens but changes it by 0.715678, matching $\sqrt{21/41}$: the bottom becomes shallower but the table rises, and the effect weakens to a square root.The separator is whether the difference of densities sits in the denominator——such a quantity grows as the difference shrinks and is sensitive to its changes, and under Paper 300 the depth of forty times and the strong dependence on density share that one root.Placed among the earlier papers——as in Paper 452 a difference is the protagonist, but there it drives a flow and here it sets a depth and becomes an amplifier. It takes the seat for groundwater that Paper 408 left in its scope note, setting sea water rising from below beside salt accumulating from above.To be honest——a sharp interface in still water was assumed, without the mixed zone, the tide or the time the interface takes to rise; flow was horizontal in the Dupuit approximation, the island a strip, and the numbers representative. No well was designed, and it is not said that wells should not be dug on islands: with the right amounts and places, the fresh water can be used.

References

1. Bear, J. (1972). *Dynamics of Fluids in Porous Media*. Elsevier.
2. Freeze, R. A., & Cherry, J. A. (1979). *Groundwater*. Prentice-Hall.
3. Todd, D. K., & Mays, L. W. (2005). *Groundwater Hydrology* (3rd ed.). Wiley.
4. Yamagishi, Y. (2026). *The Three Hundredth Census: Whether Two Things Share a Root Is Decidable* [Paper 300]. Zenodo. 10.5281/zenodo.22171914
5. Yamagishi, Y. (2026). *The Water Evaporates and the Salt Does Not, So Even Drinkable Water Kills the Land* [Paper 408]. Zenodo. 10.5281/zenodo.22699566
6. Yamagishi, Y. (2026). *What Lifts the Liquid Is Not Surface Tension but Its Difference* [Paper 452]. Zenodo. 10.5281/zenodo.22732842

Yuuki YAMAGISHI (山岸 勇輝)

© 2026 Yamagishi Complexity Torus Lab, Japan · Licensed under Creative Commons Attribution 4.0 International (CC BY 4.0)