

The Agreement of the Factors 2 Is a Case of Distinct Roots

— 2nd ed.: The First Edition Called a Coincidence to the Stand After Convicting It (the Factor-2 Problem of the Yamagishi Patch) — [Paper 58]

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— *From thinking to syncing.* —

ABSTRACT

The first edition settled the question of whether the factor **2** of Einstein's equation ($16\pi G = 2 \cdot 8\pi G$, from trace reversal) shares an origin with the **2** of the “two sheets” of the mass ladder, by **comparing the two as functions of the dimension d** . From trace reversal it obtained $f_{\text{GR}}(d) = (d-2)/(d-3)$ and from the ladder $f_{\text{ladder}} = 2$, concluding that **they meet only at $d = 4$, hence have distinct roots, and the agreement is a numerical coincidence. That conclusion is correct and the second edition does not change it.** The derivation has been re-verified: $R_{00} = \kappa_a \rho(d-3)/(d-2)$ and $f_{\text{GR}}(d) = 2 \iff d = 4$ (verified symbolically here). That the pole at $d = 3$ matches the known absence of local degrees of freedom in $2 + 1$ gravity, an independent check on the derivation, also stands. **What the second edition corrects is Section 5.** Section 4 convicted the agreement as **a numerical coincidence**. Section 5 then counted that same agreement as **“a fifth face of the specialness of $3 + 1$ dimensions”**. **These cannot both stand:** if it is a coincidence it is not evidence about the dimension, and if it is evidence it is not a coincidence. **The first edition called to the stand the very coincidence it had convicted two sections earlier** (Observation 5.1). **And the coincidence is thinner than Section 4 said.** $f_{\text{ladder}} = 2$ is **a chosen integer**. Had the sheet count been **3**, $f_{\text{GR}} = 3$ at $d = 7/2$; had it been **4**, at $d = 10/3$ (verified here). **The count is a free integer and f_{GR} takes some value at $d = 4$ whatever it is.** That $f_{\text{GR}}(4) = 2$ is remarkable only if **2** is independently significant, and the only thing making it significant here is **the chosen count itself** (Observation 5.2). **Moreover, the theorem's real content is not that two curves fail to cross.** f_{GR} is a ratio derived from the field equations and depends on d ; f_{ladder} is an integer count. **They are of different logical types** — a count does not know how many dimensions it is in (Paper 114). **Distinctness of root is settled by looking at the types, before any crossing is computed** (Observation 4.3).

Scope of this paper: NO NEW PHYSICAL LAW IS CLAIMED. Trace reversal in d dimensions and the absence of local degrees of freedom in $2 + 1$ gravity are standard (Deser, Jackiw & 't Hooft 1984). The method — asking after **the agreement of functions** rather than of values — is Paper 01's own. **Relation to the second edition of Paper 57:** That edition showed that $\kappa^2 = 16\pi G$ is merely the instruction to cancel the prefactor one started with, so **Kaluza–Klein carried the 16π and supplies no evidence at all about the factor 2. This paper therefore does not rescue a KK argument:** it settles the underlying question — whether Einstein's **2** and the ladder's **2** share a root — **on its own grounds.**

1. The question and the method

The question: Does the factor **2** in the prefactor of Einstein's equation, $16\pi G = 2 \cdot 8\pi G$ (from trace reversal), share an origin with the **2** of the “two sheets” of the mass ladder in Paper 01?

The method (Paper 01's argument, one level up): Paper 01 obtained uniqueness by asking that r^α be constant **for every r** rather than at some one point. This paper applies the same argument to the dimension: **write both factors as functions of d and ask whether they agree for every d .**

2. The Einstein-side factor $f_{\text{GR}}(d)$

Proposition 2.1 (dimensional dependence of the Einstein factor)

In a d -dimensional spacetime with $G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \kappa_d T_{\mu\nu}$, trace reversal for dust at rest gives

$$R_{00} = \kappa_d \rho \frac{d-3}{d-2} \quad (1)$$

and matching the weak field $R_{00} \simeq \nabla^2 \Phi$ to the d -dimensional Poisson equation $\nabla^2 \Phi = \Omega_{d-2} G \rho$ gives

$$\kappa_d = \Omega_{d-2} G \frac{d-2}{d-3}, \quad f_{\text{GR}}(d) := \frac{\kappa_d}{\Omega_{d-2} G} = \frac{d-2}{d-3}$$

Proof The trace gives $R(1 - d/2) = \kappa_d T$, so $R = -2\kappa_d T/(d-2)$ and $R_{\mu\nu} = \kappa_d [T_{\mu\nu} - g_{\mu\nu} T/(d-2)]$. Substituting dust, $T_{00} = \rho$, $T = -\rho$, $g_{00} = -1$, gives $R_{00} = \kappa_d \rho [1 - 1/(d-2)] = \kappa_d \rho (d-3)/(d-2)$ (verified symbolically here). At $d = 4$, $R_{00} = \frac{1}{2} \kappa \rho = 4\pi G \rho$ gives $\kappa = 8\pi G$ and $f_{\text{GR}}(4) = 2$.

Corollary 2.2 (the pole at $d = 3$ and its physical consistency, unchanged)

f_{GR} has a pole at $d = 3$, because (1) vanishes there and **no finite κ yields a Newtonian limit**. This matches the known fact that **2 + 1** gravity has no local propagating degrees of freedom (Deser, Jackiw & 't Hooft 1984) and **independently corroborates the derivation**.

f_{GR} is a derived, genuinely d -dependent ratio (verified: 2, 3/2, 4/3, 8/7, 98/97 at $d = 4, 5, 6, 10, 100$, tending to 1).

3. The ladder-side factor f_{ladder}

Proposition 3.1 (the ladder factor)

In d dimensions the shell mass is $m_d(n) = \Omega_{d-1} \int_0^1 r^{(d-1)-n} dr = \Omega_{d-1}/(d-n)$ for $d-n > 0$, so the multiplier is $S(n) = 1/(d-n)$. “Two sheets” means $S = 2$, that is $n = d - \frac{1}{2}$ (verified here). The multiplier itself is

$$f_{\text{ladder}} = 2 \quad (\text{independent of the dimension})$$

4. The verdict: distinct roots

Theorem 4.1 (distinctness of the factors 2, unchanged)

For $f_{\text{GR}}(d) = (d-2)/(d-3)$ and $f_{\text{ladder}} = 2$,

$$\frac{d-2}{d-3} = 2 \iff d-2 = 2d-6 \iff d = 4$$

so they agree only at $d = 4$. **Einstein's factor 2 and the ladder's factor 2 therefore have distinct roots, and the agreement at $d = 4$ is a numerical coincidence.**

d	$f_{\text{GR}} = (d - 2)/(d - 3)$	f_{ladder}	Agree
3	pole	2	no
4	2	2	yes
5	3/2	2	no
6	4/3	2	no
100	98/97	2	no

4.1 [Second edition] The theorem's real content is a difference of type, not a crossing

Observation 4.2 (f_{ladder} is not a function)

The first edition compared f_{GR} and f_{ladder} as **two functions** and showed that they cross only at $d = 4$. But f_{ladder} **is not a function**. It is **an integer count** — the number of sheets — and does not take d as a variable. When the first edition called it “constant”, the accurate statement is not that it is **a function independent of d** but that **it is not a function of d at all**.

Observation 4.3 (the types settle it before the crossing is computed)

The real content of Theorem 4.1 is therefore **a difference of logical type**:

What it is		Does it know d ?
$f_{\text{GR}}(d)$	a ratio derived from the field equations	yes, as $(d - 2)/(d - 3)$
f_{ladder}	a count, that is an integer	no

A count does not know how many dimensions it is in (the measure of Paper 114). **Distinctness of root is settled by looking at the types, before any crossing is computed**; the computation merely confirms it.

This is the stronger statement. “Two functions cross at only one point” might have come out otherwise. **“One knows d and the other does not” is a statement about structure.**

5. [Second edition] Withdrawing the first edition's Section 5

The first edition's Section 5 argued that since the two factors agree only at $d = 4$ (spatial three dimensions), “the agreement of the two 2s” is a **fifth face** of the $d = 3$ selection collected in Paper 56; and that the answer to “why did they appear to agree?” is “**because we live in $3 + 1$ dimensions**”.

Observation 5.1 (Section 5 calls to the stand what Section 4 convicted)

The two sections cannot both stand.

Section 4	the agreement is a numerical coincidence (distinct roots)
Section 5	the agreement is evidence for the specialness of $3 + 1$ (a fifth face)

If it is a coincidence it cannot be evidence about the dimension; if it is evidence it is not a coincidence. What the first edition had itself pronounced a numerical coincidence, it counted two sections later as **a member of the dimensional selection**.

Observation 5.2 (the coincidence is thinner than Section 4 said)

$f_{\text{ladder}} = 2$ is a **chosen integer**. A different count moves the crossing (verified here):

Count	d at which $f_{\text{GR}}(d)$ takes that value
2	4
3	7/2
4	10/3
5	13/4

The count is a free integer, and f_{GR} takes some value at $d = 4$ whatever it is. That $f_{\text{GR}}(4) = 2$ is remarkable only if 2 is independently significant, and what makes it significant here is **the chosen count itself**.

Corollary 5.3 (the fifth face is withdrawn)

The facts collected in Paper 56 for the selection of $d = 3$ — orbital stability, atomic stability, wave propagation — are all statements about **quantities that know d** . **A count cannot be placed beside them.** In the measure of Paper 114, **an object that does not know the dimension cannot stand on the side that selects it.**

The designation “fifth face” is therefore withdrawn. What remains is the verdict of Section 4 alone: **distinct roots, and the agreement at $d = 4$ is a numerical coincidence — into which no further meaning is to be read.**

6. Conclusion

1. The Einstein-side factor is $f_{\text{GR}}(d) = (d - 2)/(d - 3)$ (Proposition 2.1, re-verified symbolically): from trace reversal, equal to **2** at $d = 4$ and with a pole at $d = 3$.
2. **The pole at $d = 3$ matches the known absence of local degrees of freedom in $2 + 1$ gravity and independently corroborates the derivation** (Corollary 2.2, unchanged).
3. The ladder-side multiplier is $f_{\text{ladder}} = 2$, independent of the dimension (Proposition 3.1).
4. **The two agree only at $d = 4$; the factors 2 therefore have distinct roots and the agreement is a numerical coincidence** (Theorem 4.1, unchanged).
5. [2nd ed.] **The theorem's real content is a difference of logical type, not a crossing — f_{GR} is a derived d -dependent ratio and f_{ladder} an integer count, and a count does not know how many dimensions it is in.** The distinctness is settled before the crossing is computed (Observations 4.2, 4.3).
6. [2nd ed.] **The first edition's Section 5 (“the fifth face”) is withdrawn** — what Section 4 pronounced a numerical coincidence, Section 5 counted as evidence for the dimension. **These cannot both stand** (Observation 5.1). And $f_{\text{ladder}} = 2$ is a chosen integer: counts of **3, 4, 5** move the crossing to **7/2, 10/3, 13/4** — **what gives the 2 its significance is the chosen count itself** (Observation 5.2, Corollary 5.3).
7. [2nd ed.] **The second edition of Paper 57 changes this paper's standing** — the KK charge normalisation merely carried the **16 π** and gives no evidence about the factor **2**. **This paper therefore does not rescue a KK argument; it settles the underlying question on its own grounds.**

Closing: With a single move — asking after the agreement of functions rather than of values — the first edition settled the identity of the factors **2** as a case of distinct roots. **That verdict is correct.** What the second edition adds is **keeping to it:** what one has pronounced a coincidence must not be counted, two sections later, as something meaningful. **One must not push with the hand that applied the brake.**

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