The Age at Which a Forest Takes Up Carbon Fastest Is Not the Age at Which It Holds the Most

— the rate of uptake peaks at **27.465307** years and the mean rate at **47.595342** years, while the stock is still growing at 120 years — a 120-year-old forest takes up carbon at only **0.054640** of the peak rate — [Paper 581]

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— *From thinking to syncing.* —

Abstract

Because forests take up carbon dioxide, “protecting forests” and “growing forests” are easily taken to point the same way.What this paper shows is that the rate at which a forest takes up carbon, the mean rate, and the amount it holds each peak at a different age: young forests take up, old forests hold.No new theorem or law is claimed. First, the rate of uptake peaks at 27.465307 years——the closed form $\ln 3/k$ and the numerical peak agree, giving 8.8889 m^3 /ha/yr, or 8.4741 t of carbon dioxide per ha per year (Section 2). **Second, the mean rate of uptake peaks at 47.595342 years——there the rate and its mean agree at 6.474337 m^3/ha/yr, or 6.1722 t of carbon dioxide per ha per year** (Section 3). **Third, the stock keeps growing——0.975513 of the ceiling at 120 years, 5.841932 times a 20-year-old forest, while the rate of uptake is only 0.054640 of its peak** (Section 4). **Fourth, and this is the core.Three peaks lie at three ages——the rate a little over 27 years, the mean rate just under 48, and the stock endlessly later. Young forests take up; old forests hold** (Section 4). **Fifth, the peak of the mean is flat——felling at 40 years still gives 0.981505 of it, and at 55 years 0.987201; yet at 60 years the stock is 0.751789 of the ceiling and the rate 0.506284 of its peak, both moving a great deal** (Section 5). **Sixth, the separator is flow against stock——felling and replanting at the peak of the mean maximizes the average rate of uptake, but drops the average stock to 0.254899 of the ceiling** (Section 5). **Keywords**: forest carbon, Chapman-Richards growth curve, current annual increment, mean annual increment, stand age.

Scope of this paper (scope note): No new theorem or law is claimed——the Chapman--Richards growth curve and the relation between current and mean annual increment are standard.The growth curve is representative——a ceiling of 500 m^3/ha, a rate constant of 0.04 per year and an exponent of 3; species and site change these greatly.The conversion to carbon is representative——wood density 0.4 t/m^3, a factor of 1.3 for branches, leaves and roots, and a carbon fraction of 0.5.Carbon in harvested wood is not included——if felled timber lasts long in houses and furniture, its carbon stays held; this paper counts only the stock in the forest.Soil carbon, mortality, thinning and disturbance are not included.No forestry policy is recommended——"fell or protect" is not answered; what is shown is that the answer changes with what is maximized.Relation to earlier papers: Paper 547 showed that the optimum order quantity is flat around itself, so it need not be held exactly——the peak of mean increment is flat in the same way; this paper sets beside it the large movement of other quantities near that flat one.Paper 386 showed that "hard to dissolve" points at two quantities, solubility and rate of dissolution——"a forest that takes up well" also points at two, rate and stock.Paper 300 showed that sharing a root is decidable——the three peaks come from one growth curve and share a root, but measure different quantities.What is added is confirming the peak of the rate against the closed form $\ln 3/k$ numerically, confirming that the rate equals its mean at the peak of the mean, setting out how the rate falls while the stock goes on growing, obtaining the average stock over a rotation by a closed form and by numerical integration, and placing the separator on the distinction between flow and stock.

1. Notation and the Question

Setting: the volume of a stand (the stock) is written with the Chapman--Richards growth curve

$$V(t) = V_{\max}\left(1 - e^{-kt}\right)^3.$$

The rate of uptake is the current annual increment $V'(t)$, and the mean rate over a rotation of felling and replanting is the mean annual increment $V(t)/t$.Volume is converted to carbon dioxide by wood density, a factor for branches, leaves and roots, the carbon fraction and $44/12$.

Observation 1.1

forests take up carbon dioxide.So "protecting forests" and "growing forests" are easily taken to point the same way——but a forest that takes up well and a forest that holds much need not be the same forest.

Question: at what ages do the rate of uptake, the mean rate and the stock each peak?

2. The Peak of the Rate

Computation 2.1: with a ceiling of 500 m^3/ha and $k = 0.04$ per year, the peak of $V'(t)$.The closed form $\ln 3/k = \mathbf{27.465307}$ years agrees with the numerically found peak to 10^{-4} years.

> **Observation 2.2**
>
> the rate of uptake is greatest in a young forest.At the peak it is $\mathbf{8.8889}$ m^3/ha/yr, or $\mathbf{8.4741}$ t of carbon dioxide per ha per year——a forest under thirty years old, not yet at half its ceiling, takes up fastest.

3. The Peak of the Mean Rate

Computation 3.1: the peak of the mean increment $V(t)/t$, found by solving $V'(t)t = V(t)$.It lies at $\mathbf{47.595342}$ years, where the rate and its mean agree at $\mathbf{6.474337}$ m^3/ha/yr (to 10^{-9}).

> **Observation 3.2**
>
> the peak of the mean comes twenty years after the peak of the rate.It is $\mathbf{6.1722}$ t of carbon dioxide per ha per year——if felling and replanting are repeated, felling at this age takes up the most per year on average.

4. The Stock

Computation 4.1: stock and rate of uptake by age.

age	stock (m^3/ha)	of the ceiling	rate (m^3/ha/yr)	of the peak
20 yr	**83.4924**	**0.166985**	**8.1752**	**0.919714**
40 yr	**254.1837**	**0.508367**	**7.7161**	**0.868063**
60 yr	**375.8944**	**0.751789**	**4.5003**	**0.506284**
80 yr	**441.3152**	**0.882630**	**2.2504**	**0.253171**
120 yr	**487.7567**	**0.975513**	**0.4857**	**0.054640**
200 yr	**499.4970**	**0.998994**	**0.0201**	**0.002263**

Observation 4.2

this is the core.The rate peaks a little after 27 years, the mean rate just before 48, and the stock endlessly later——three peaks at three ages. Young forests take up; old forests hold.

Observation 4.3

an old forest takes up almost nothing and holds a great deal.A 120-year-old forest holds **5.841932** times as much as a 20-year-old one, yet takes up at only 0.054640 of the peak rate——neither "a forest that no longer takes up is useless" nor "a forest that holds most is best" follows; the quantities measured differ.

5. Felling and Replanting

Computation 5.1: the average stock over a rotation felled and replanted at the peak of the mean, 47.595342 years.The closed form and numerical integration agree to 10^{-6}: **127.4494** m^3/ha, or **0.254899** of the ceiling.The mean rate of uptake over that rotation is **6.4743** m^3/ha/yr.

Observation 5.2

maximizing the mean rate cuts the mean stock to a quarter.An aged forest takes up almost nothing but holds close to its ceiling——felling and replanting keeps the rate up, but what can be held in the forest falls to about a quarter of the ceiling.

Computation 5.3: the felling age moved either side of the peak of the mean.

felling age	mean rate (of the peak)
35 yr	**0.943602**
40 yr	**0.981505**
55 yr	**0.987201**
60 yr	**0.967652**

Observation 5.4

the peak of the mean is flat.Moving the felling age from 40 to 55 years never drops the mean rate below 98 per cent of its peak——yet at 60 years the stock is 0.751789 of the ceiling and the current rate 0.506284 of its peak: other quantities move a great deal. Watching only the flat one hides those that move.

6. The Separator

Observation 6.1

the separator is flow against stock.The rate of uptake is the slope of the growth curve (a flow); the stock is its height——the flow peaks partway along the curve, the stock grows to the end. Which is maximized changes the answer to the best age.

Observation 6.2

flat around the optimum, as in Paper 547.There, doubling the order quantity raised the total cost only to 1.25 times——here, shifting the felling age hardly lowers the mean rate. What this paper adds is that beside that flat quantity, the stock and the current rate move a great deal.

Observation 6.3

the shape of Paper 386.There, “hard to dissolve” pointed at two quantities, solubility and rate of dissolution ——here, “a forest that takes up well” points at two, rate and stock.

Observation 6.4

one root under Paper 300, different quantities.All three peaks come from one growth curve——what differs is whether the slope, the mean of the slope, or the height is looked at.

> **Observation 6.5**
>
> **what is not said.No forestry policy is recommended——with carbon in harvested wood left out, “fell or protect” cannot be answered here. What is shown is that the answer changes with what is maximized.**

7. Roll Call and Closing

1. **the rate peaks at 27.465307 years** (Computation 2.1): **matching $\ln 3/k$.**
2. **8.4741 t CO_2/ha/yr at the peak** (Observation 2.2): **young forests are fastest.**
3. **the mean peaks at 47.595342 years** (Computation 3.1): **rate and mean agree at 6.474337.**
4. **0.975513 of the ceiling at 120 years** (Computation 4.1): **the stock keeps growing.**
5. **three peaks at three ages** (Observation 4.2): **the core.**
6. **5.841932 times the stock, 0.054640 of the rate at 120 years** (Observation 4.3): **different quantities.**
7. **mean stock over a rotation 0.254899 of the ceiling** (Computation 5.1): **the price of keeping the rate up.**
8. **0.981505 at 40 years, 0.987201 at 55** (Computation 5.3): **the peak of the mean is flat.**
9. **the separator is flow or stock** (Observation 6.1): **slope or height.**
10. **flat, as in Paper 547** (Observation 6.2): **other quantities move beside it.**

Closing: **Because forests take up carbon dioxide, “protecting forests” and “growing forests” are easily taken to point the same way.Yet the rate of uptake, the mean rate and the stock each peak at a different age——under Chapman--Richards growth the rate peaks, as the closed form $\ln 3/k$ says, at 27.465307 years, at 8.4741 t of carbon dioxide per ha per year. The mean rate peaks at 47.595342 years, where rate and mean agree at 6.474337 m^3/ha/yr. The stock goes on growing, to 0.975513 of its ceiling at 120 years and 5.841932 times a 20-year-old forest, while the rate then is only 0.054640 of its peak.Young forests take up; old forests hold——felling and replanting at the peak of the mean maximizes the average rate of uptake, but the average stock held in the forest falls to 0.254899 of the ceiling. The peak of the mean is flat, still 0.981505 at 40 years and 0.987201 at 55, yet at 60 years the stock is 0.751789 of the ceiling and the rate 0.506284 of its peak, both moving a great deal.The separator is flow against stock——the rate of uptake is the slope of the growth curve and the stock its height; the flow peaks partway, the stock grows to the end. Under Paper 300 the three peaks share the root of one growth curve but measure different quantities.Placed among the earlier papers——as in Paper 547 the optimum is flat around itself, and this paper sets beside it the other quantities that move a great deal. As “hard to dissolve” pointed at two quantities in Paper 386, “a forest that takes up well” points at two.To be honest——the growth curve and the conversion to carbon are representative, and carbon in harvested wood, soil carbon, mortality, thinning and disturbance are left out. No forestry policy is recommended: what is shown is that the best age changes with what is maximized.**

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