What Sets the Series of Overtones Is Not the Closed End but How the Tube Widens from It

— with one end closed, a cylinder has only odd harmonics and its second resonance a twelfth above, while a cone closed at its apex has every harmonic and an octave — between them, the distance x_0 from the closed end to the apex connects the two through $\tan kL = -kx_0$ — [Paper 582]

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— *From thinking to syncing.* —

Abstract

The overtones of wind instruments are often explained by the ends: a tube closed at one end has odd harmonics, a tube open at both has them all.What this paper shows is that among tubes closed at one end, a cylinder has only odd harmonics and a cone has them all, and that the distance from the closed end to the apex connects the two continuously through one formula.No new theorem or law is claimed. First, the two limits of a cylinder——at 0.60 m, a cylinder open at both ends has its fundamental at 285.8333 Hz with harmonics 1, 2, 3; closed at one end, 142.9167 Hz with harmonics 1, 3, 5 (Section 2). **Second, a cone closed at its apex sounds like an open cylinder though it is closed——its fundamental is 285.8333 Hz and the ratio to its second resonance is exactly 2.000000** (Section 3). **Third, and this is the core.Varying the distance x_0 from the closed end to the apex moves the ratio to the second resonance continuously from 2 to 3——2.012230 at $x_0/L = 0.1$, 2.111700 at 0.3, 2.421768 at 1 and 2.719873 at 3** (Section 4). **Fourth, a tube in between does not leap by a pure interval——at $x_0/L = 0.3$ the second resonance is 1294.0857 cents above, neither an octave (1200) nor a twelfth (1901.955001)** (Section 4). **Fifth, the fundamental moves too——at the same length it falls from 272.3177 Hz at $x_0/L = 0.05$ to 159.9641 Hz at 3** (Section 4). **Sixth, the separator is how the tube widens as seen from its closed end——not whether an end is closed, but whether the cross-section widens near the closed end or stays constant, decides the series** (Section 5). **Keywords**: wind instruments, conical bore, overtones, resonance, reed instruments.

Scope of this paper (scope note): **No new theorem or law is claimed——the resonances of cylindrical and conical tubes are all standard.Only ideal tubes are treated——rigid walls, no losses, and no end correction (the short length by which the air column extends beyond an open end).The volume of a reed or mouthpiece is not included——in a real instrument the volume near the closed end shifts the resonances slightly.The numbers are representative——a length of 0.60 m and a speed of sound of 343 m/s.No instrument is designed——nor is any instrument's range given; only the pattern is used that a cylindrical tube (clarinet) leaps a twelfth and a conical one (saxophone, oboe) an octave.Relation to earlier papers**: **Paper 299 showed that a drum lacks pitch because of its boundary, not its equation, and set musical instruments aside in its scope note——this paper takes that seat. The root, that the shape of the boundary chooses the series of overtones, is shared; what is added is that one number x_0 connects the two limits of cylinder and cone continuously.Paper 466 counted in cents that all intervals cannot be tuned at once——the second resonance of a tube in between lands on none of its pure intervals.Paper 300 showed that sharing a root is decidable——"a cylinder has odd harmonics" and "a cone has them all" are the two ends of one formula, $\tan kL = -kx_0$, and share a root.**What is added is **placing the resonance condition of a closed truncated cone as $\tan kL = -kx_0$ and checking its two limits (ratios 2 and 3) numerically**, setting out how the ratio to the second resonance and the fundamental move in between, and **placing the separator on how the tube widens from its closed end**.

1. Notation and the Question

Setting: **in a conical tube, the pressure at distance x from the apex takes the form $p = (A/x)\sin(kx + \phi)$.Closing the narrow end (at x_0 from the apex) and opening the wide end (a length L further on), with no air motion at the closed end and zero pressure at the open end, gives**

$$\tan kL = -k\,x_0.$$

$x_0 \to 0$ is a cone closed at its apex, and $x_0 \to \infty$ the limit of a cylinder.

Observation 1.1

the overtones of wind instruments are often explained by whether the ends are closed.A tube closed at one end has odd harmonics, a tube open at both has them all——yet among instruments closed at one end by a reed, some have only odd harmonics and some have them all.

Question: **what sets the series of overtones of a tube closed at one end?**

2. Two Cylinders

Computation 2.1: **length 0.60 m, speed of sound 343 m/s, no end correction.**

tube	fundamental	harmonics
cylinder open at both ends	**285.8333** Hz	**1, 2, 3**
cylinder closed at one end	**142.9167** Hz	**1, 3, 5**

> **Observation 2.2**
>
> **a cylinder closed at one end sounds an octave lower and leaps a twelfth.Its second resonance is three times the fundamental, $1200\log_2 3 = \mathbf{1901.955001}$ cents above——this is why an instrument with a cylindrical tube (the clarinet) leaps a twelfth, not an octave, when overblown.**

3. A Closed Cone

Computation 3.1: **for a cone closed at its apex ($x_0 \to 0$) the condition is $\tan kL = 0$, with resonances at $kL = \pi, 2\pi, \ldots$.Numerically $kL = \mathbf{3.141593}$ and $\mathbf{6.283185}$, a ratio of exactly $\mathbf{2.000000}$ (to 10^{-6}).**

> **Observation 3.2**
>
> **closed, yet it sounds like an open cylinder.Its fundamental, $\mathbf{285.8333}$ Hz, matches that of a cylinder of the same length open at both ends to 10^{-6}, and every harmonic is present——this is why instruments with conical tubes (the saxophone, the oboe) leap an octave. Being closed at one end was not the reason for odd harmonics.**

4. Connecting the Two

Computation 4.1: **the distance x_0 from the closed end to the apex was varied and the first two roots of $\tan kL = -kx_0$ found by bisection.**

x_0/L	kL (first, second)	fundamental	ratio to second	cents
0.05	**2.993043**, **5.992091**	**272.3177** Hz	**2.002006**	**1201.7360**
0.1	**2.862773**, **5.760558**	**260.4653** Hz	**2.012230**	**1210.5546**
0.3	**2.498399**, **5.275869**	**227.3133** Hz	**2.111700**	**1294.0857**
1	**2.028758**, **4.913180**	**184.5836** Hz	**2.421768**	**1531.2727**
3	**1.758164**, **4.781983**	**159.9641** Hz	**2.719873**	**1732.2469**

Observation 4.2

this is the core.The ratio to the second resonance moves monotonically and continuously from 2 to 3 as x_0 grows——in the limit of a cylinder ($x_0/L = 10^6$) the ratio is $\mathbf{2.999999}$, with $kL = \mathbf{1.570797}$ and $\mathbf{4.712389}$, the odd series again. Cylinder and cone are not separate types but the two ends of one formula.

Observation 4.3

a tube in between does not leap by a pure interval.At $x_0/L = 0.3$ the second resonance lies 1294.0857 cents above, neither an octave nor a twelfth——in a tube whose overtones are not integer multiples, the leaps on overblowing do not line up as intervals. Usable tubes lean towards the cone or the cylinder for this reason.

Observation 4.4

the fundamental moves too.At the same length of 0.60 m it falls from 272.3177 Hz at $x_0/L = 0.05$ to 159.9641 Hz at 3——the widening sets not only the series but the pitch.

5. The Separator

Observation 5.1

the separator is how the tube widens as seen from its closed end.In a cone widening from the closed end, the pressure weakens quickly near that end, which then plays the part of an open end——in a cylinder of constant section the closed end stays a closed end all the way, and only odd harmonics remain. Not whether an end is closed, but how the tube widens from it, decides the series.

Observation 5.2

one root with Paper 299.It showed that a drum lacks pitch because of its boundary, not its equation, and set instruments aside in its scope note——this paper takes that seat. The shape of the boundary choosing the series is the shared root; what is added is that one number x_0 connects cylinder and cone continuously.

Observation 5.3

none of the pure intervals of Paper 466.It counted in cents that stacking fifths misses the octave and that all intervals cannot be tuned at once——the second resonance of a tube in between lands on no pure interval at all.

Observation 5.4

one root, under Paper 300.“A cylinder has odd harmonics” and “a cone has them all” are the $x_0 \to \infty$ and $x_0 \to 0$ ends of the one formula $\tan kL = -kx_0$——what looked like two separate rules were the two ends of one formula.

Observation 5.5

what is not said.No instrument is designed, and no instrument’s range is given——what is shown is that the series of overtones is set not by the closed end but by how the tube widens from it.

6. Roll Call and Closing

1. **cylinder open at both ends 285.8333 Hz** (Computation 2.1): **all harmonics.**
2. **cylinder closed at one end 142.9167 Hz** (Computation 2.1): **odd harmonics only.**
3. **a twelfth is 1901.955001 cents** (Observation 2.2): **where a cylinder leaps.**
4. **closed cone ratio 2.000000** (Computation 3.1): **closed, yet like an open cylinder.**
5. **2.012230 at $x_0/L = 0.1$** (Computation 4.1): **slightly off.**
6. **2.421768 at $x_0/L = 1$** (Computation 4.1): **halfway.**
7. **2.999999 in the cylinder limit** (Observation 4.2): **the core, the two ends of one formula.**
8. **1294.0857 cents at $x_0/L = 0.3$** (Observation 4.3): **no pure interval.**
9. **fundamental from 272.3177 to 159.9641 Hz** (Observation 4.4): **the pitch moves too.**
10. **the separator is the widening from the closed end** (Observation 5.1): **not whether it is closed.**

Closing: **The overtones of wind instruments are often explained by whether the ends are closed.Yet among tubes closed at one end, the series of overtones changes with how the tube widens——at 0.60 m a cylinder closed at one end has its fundamental at 142.9167 Hz and only odd harmonics, leaping a twelfth, 1901.955001 cents, when overblown. Closed at its apex instead, a cone has the fundamental of a cylinder open at both ends, 285.8333 Hz, with every harmonic present and a ratio to its second resonance of exactly 2.000000.Between them, the distance x_0 from the closed end to the apex connects the two continuously through $\tan kL = -kx_0$——the ratio to the second resonance rises monotonically, 2.012230 at $x_0/L = 0.1$, 2.111700 at 0.3, 2.421768 at 1 and 2.719873 at 3, returning to 2.999999 in the cylinder limit. A tube in between leaps 1294.0857 cents at 0.3, neither octave nor twelfth, and its fundamental moves from 272.3177 to 159.9641 Hz. Usable tubes lean to cone or cylinder in order to keep overtones in integer multiples.The separator is how the tube widens as seen from its closed end——in a widening cone the pressure weakens quickly near the closed end, which then plays the part of an open end; in a cylinder of constant section the closed end stays closed all the way, and only odd harmonics remain. Under Paper 300 the two types are the ends of one formula and share a root.Placed among the earlier papers——as in Paper 299 the shape of the boundary chooses the series, and this paper takes the seat for instruments that its scope note left, connecting the two limits by one number. The leap of a tube in between lands on none of the pure intervals of Paper 466.To be honest——these are ideal tubes, with rigid walls and no losses, end correction or reed volume. Length and speed of sound are representative, no instrument was designed, and no instrument's range is given. What is shown is that the series is set not by the closed end but by how the tube widens from it.**

References

1. Benade, A. H. (1976). *Fundamentals of Musical Acoustics*. Oxford University Press.

2. Fletcher, N. H., & Rossing, T. D. (1998). *The Physics of Musical Instruments* (2nd ed.). Springer.

3. Rayleigh, J. W. S. (1877). *The Theory of Sound*. Macmillan.

4. Yamagishi, Y. (2026). *A Drum Has No Pitch Because of Its Boundary, Not Its Equation* [Paper 299]. Zenodo. 10.5281/zenodo.22171877

5. Yamagishi, Y. (2026). *The Three Hundredth Census: Whether Two Things Share a Root Is Decidable* [Paper 300]. Zenodo. 10.5281/zenodo.22171914

6. Yamagishi, Y. (2026). *That Everything Cannot Be Tuned at Once Is a Proved Fact* [Paper 466]. Zenodo. 10.5281/zenodo.22733284

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