More Damping in a Base Isolator Transmits More Shaking on One Side

— a base-isolated building of period **4** seconds shaken at a period of **1** second receives **0.071777** of the ground's shaking with damping **0.05**, but **0.218589** with damping **0.40** — damping is needed all the same, because in resonance with long-period shaking it holds the isolator's displacement down — [Paper 584]

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— *From thinking to syncing.* —

Abstract

More damping is easily taken to mean less shaking.What this paper shows is that on the side where the ratio of ground frequency to a building's natural frequency exceeds $\sqrt{2}$, more damping passes more shaking to the building, and the effect of damping reverses at $\sqrt{2}$.No new theorem or law is claimed. First, resonance amplifies shaking——a building of natural period 1 second shaken at a period of 1 second receives 10.049876 times the ground's shaking with damping 0.05 (Section 2). **Second, isolating the building to a period of 4 seconds cuts it to 0.071777——the frequency ratio becomes 4 and little shaking passes** (Section 4). **Third, at a frequency ratio of $\sqrt{2}$ the transmitted shaking is exactly 1, whatever the damping——damping 0.05, 0.20, 0.40 and 1.00 all give 1 to 10^{-12}** (Section 3). **Fourth, and this is the core.Beyond $\sqrt{2}$, more damping passes more shaking——at a ratio of 4, damping 0.05, 0.20 and 0.40 give 0.071777, 0.125077 and 0.218589; inside it, at a ratio of 1, damping instead cuts 10.049876 to 1.600781** (Section 4). **Fifth, there is still a reason for damping——when an isolated building resonates with long shaking of period 4 seconds, the isolator's displacement is $1/(2\zeta)$: 10.000000 times with damping 0.05, held to 1.250000 with 0.40; under ordinary shaking the displacement barely changes, from 1.066288 to 1.043192** (Section 5). **Sixth, the separator is whether the frequency ratio lies above or below $\sqrt{2}$——the same damping reduces shaking below and increases it above** (Section 6). **Keywords**: base isolation, damping, transmissibility, frequency ratio, ground motion.

Scope of this paper (scope note): **No new theorem or law is claimed——single-degree-of-freedom vibration under ground motion and its transmissibility are standard.One degree of freedom and linear viscous damping only——the building is one mass on the isolators; the non-linear behaviour of isolation bearings, lead hysteresis and torsion are not included.Shaking has a single period——real earthquakes contain many periods; this paper takes them one at a time.The periods are representative——a natural period of 1 second for an ordinary building and 4 seconds for an isolated one, and ground shaking at 1 and 4 seconds.No building is designed——how much damping to use is not stated.Relation to earlier papers**: **Paper 351 showed that standing and sitting on a swing are two mechanisms, and set damping aside in its scope note——this paper takes that seat and makes its subject the reversal of damping's effect with the frequency ratio.Paper 557 showed that the angle that strengthens a tooth points against the angle that smooths the drive——here too one knob, damping, carries two demands pointing opposite ways, the shaking transmitted and the isolator's displacement.Paper 300 showed that sharing a root is decidable——"inside $\sqrt{2}$ damping reduces shaking" and "outside it increases it" come from one transmissibility formula and share a root.**What is added is **checking with four dampings that the transmitted shaking is exactly 1 at a ratio of $\sqrt{2}$, and showing why from the form of the formula, setting out that damping increases shaking outside and reduces it inside, checking against the closed form that in long-period resonance the displacement is held to $1/(2\zeta)$,** and **placing the separator at the ratio $\sqrt{2}$.**

1. Notation and the Question

Setting: **a building of natural period T_n and damping ratio ζ is shaken by the ground at period T.With the frequency ratio $r = T_n/T$ (ground frequency over the building's natural frequency), the ratio of the building's acceleration to the ground's, and of the isolator's displacement to the ground's, are**

$$\mathrm{TR} = \sqrt{\frac{1+(2\zeta r)^2}{(1-r^2)^2+(2\zeta r)^2}}, \qquad \frac{Z}{Y} = \frac{r^2}{\sqrt{(1-r^2)^2+(2\zeta r)^2}}.$$

Observation 1.1

more damping is easily taken to mean less shaking.Shaking at resonance is indeed smaller the more damping there is——whether that holds at every frequency ratio is another question.

Question: **does more damping always pass less shaking to the building?**

2. Transmitted Shaking

Computation 2.1: **the transmitted acceleration ratio for several frequency ratios and dampings.**

ratio r	damping 0.05	damping 0.20	damping 0.40
1	**10.049876**	**2.692582**	**1.600781**
2	**0.339182**	**0.412461**	**0.554940**
4	**0.071777**	**0.125077**	**0.218589**

> **Observation 2.2**
>
> **at resonance ($r = 1$) more damping means less shaking.A building of natural period 1 second shaken at 1 second receives 10.049876 times the ground's shaking with damping 0.05, and 1.600781 with 0.40——so far, as intuition says.**

> **Observation 2.3**
>
> **yet in the rows for $r = 2$ and $r = 4$, more damping means more shaking.From 0.339182 to 0.554940 at a ratio of 2, and from 0.071777 to 0.218589 at 4——the same damping works in opposite directions in different rows.**

3. The Boundary Is $\sqrt{2}$

Computation 3.1: **at $r = \sqrt{2}$, damping set to 0.05, 0.20, 0.40 and 1.00.All four give 1.000000000000, matching 1 to 10^{-12}.**

> **Observation 3.2**
>
> **at $\sqrt{2}$, damping has no effect at all.When $r^2 = 2$, $(1 - r^2)^2 = 1$, so the damping term $(2\zeta r)^2$ is added to the same 1 above and below, and the ratio is exactly 1——inside $\sqrt{2}$, $(1 - r^2)^2$ is less than 1, so adding the same damping term raises the denominator by the larger fraction and the ratio falls; outside, $(1 - r^2)^2$ exceeds 1, so the numerator rises by the larger fraction and the ratio grows.**

4. An Isolated Building Under Ordinary Shaking

Computation 4.1: **an isolated building of natural period 4 seconds shaken at the ordinary period of 1 second ($r = 4$).**

damping	transmitted shaking	isolator displacement
0.05	**0.071777**	**1.066288**
0.20	**0.125077**	**1.060650**
0.40	**0.218589**	**1.043192**

> **Observation 4.2**
>
> **this is the core.In an isolated building, raising the damping from 0.05 to 0.40 raises the transmitted shaking from 0.071777 to 0.218589——isolation reduces shaking by lengthening the period and moving outside $\sqrt{2}$, and there damping becomes a path for shaking to pass. The isolator's displacement barely falls, from 1.066288 to 1.043192.**

5. Long-Period Shaking

Computation 5.1: **the same isolated building shaken at a long period of 4 seconds ($r = 1$, resonance).**

damping	transmitted shaking	isolator displacement $(1/(2\zeta))$
0.05	**10.049876**	**10.000000**
0.20	**2.692582**	**2.500000**
0.40	**1.600781**	**1.250000**

> **Observation 5.2**
>
> **damping is still needed, for long-period shaking.At resonance the displacement ratio matches the closed form $1/(2\zeta)$ to 10^{-12}: ten times with damping 0.05, held to 1.25 with 0.40——the isolators can move only so far, so with too little damping a long shaking drives them into their stops. Damping is a knob that loses a little under ordinary shaking and gains a great deal under long shaking.**

6. The Separator

Observation 6.1

the separator is whether the frequency ratio lies above or below $\sqrt{2}$.Below, damping reduces shaking; above, it increases it——“more damping, less shaking” holds only inside $\sqrt{2}$.

Observation 6.2

taking the seat reserved by Paper 351.It showed that pumping a swing is two mechanisms, parametric excitation at twice the natural frequency and forced oscillation at once, and set damping aside in its scope note——this paper, on the side of forced oscillation, makes the reversal of damping’s effect with the frequency ratio its subject.

Observation 6.3

the shape of Paper 557.There, the angle that strengthened a tooth pointed against the angle that smoothed the drive——here one knob, damping, carries two demands pointing opposite ways: not passing ordinary shaking, and not moving too far under long shaking.

Observation 6.4

one root, under Paper 300.“Inside, damping reduces shaking” and “outside, it increases it” come from one transmissibility formula——what separates them is only whether $(1-r^2)^2$ in the formula is above or below 1.

Observation 6.5

what is not said.No building is designed, and how much damping to use is not stated——what is shown is that damping’s effect reverses with the frequency ratio, at a boundary of $\sqrt{2}$.

7. Roll Call and Closing

1. **10.049876 times at resonance** (Computation 2.1): **period 1 second, damping 0.05.**
2. **damping cuts resonance to 1.600781** (Observation 2.2): **as intuition says.**
3. **exactly 1 at $\sqrt{2}$ whatever the damping** (Computation 3.1): **four dampings to 10^{-12}.**
4. **the reason is $(1-r^2)^2=1$** (Observation 3.2): **numerator and denominator align.**
5. **0.071777 with isolation** (Computation 4.1): **period 4 seconds.**
6. **more damping raises it to 0.218589** (Observation 4.2): **the core.**
7. **displacement from 1.066288 to 1.043192** (Computation 4.1): **barely falls.**
8. **long-period resonance: displacement from 10.000000 to 1.250000** (Computation 5.1): **why damping is still needed.**
9. **the separator is the ratio $\sqrt{2}$** (Observation 6.1): **reducing below, increasing above.**
10. **the seat reserved by Paper 351** (Observation 6.2): **damping as the subject.**

Closing: **More damping is easily taken to mean less shaking.Near resonance that is right——a building of natural period 1 second shaken at 1 second receives 10.049876 times the ground's shaking with damping 0.05, and 1.600781 with 0.40.Beyond a frequency ratio of $\sqrt{2}$, however, more damping passes more shaking——at $\sqrt{2}$ the ratio is exactly 1 whatever the damping, because $(1-r^2)^2$ there is exactly 1 and numerator and denominator align. An isolated building of period 4 seconds shaken at 1 second receives 0.071777 with damping 0.05 but 0.218589 with 0.40, while the isolator's displacement barely falls, from 1.066288 to 1.043192. Damping is still needed for long-period shaking: when the same building resonates at 4 seconds, the isolator's displacement is $1/(2\zeta)$, 10.000000 times with damping 0.05 and 1.250000 with 0.40.The separator is whether the frequency ratio lies above or below $\sqrt{2}$——below, damping reduces shaking; above, it becomes a path for shaking to pass. Under Paper 300 both come from one transmissibility formula and share a root.Placed among the earlier papers——this takes the seat for damping that Paper 351 left in its scope note. As in Paper 557, one knob carries two demands pointing opposite ways.To be honest——this is one degree of freedom, linear viscous damping and a single period of shaking, without the non-linearity of isolation bearings, torsion or the many periods of a real earthquake. The periods are representative, no building was designed, and how much damping to use is not stated.**

References

1. Chopra, A. K. (2012). *Dynamics of Structures: Theory and Applications to Earthquake Engineering* (4th ed.). Prentice Hall.
2. Den Hartog, J. P. (1956). *Mechanical Vibrations* (4th ed.). McGraw-Hill.
3. Kelly, J. M. (1997). *Earthquake-Resistant Design with Rubber* (2nd ed.). Springer.
4. Yamagishi, Y. (2026). *The Three Hundredth Census: Whether Two Things Share a Root Is Decidable* [Paper 300]. Zenodo. 10.5281/zenodo.22171914

5. Yamagishi, Y. (2026). *Standing and Sitting on a Swing Are Two Mechanisms* [Paper 351]. Zenodo. 10.5281/zenodo.22695649

6. Yamagishi, Y. (2026). *The Angle That Strengthens a Tooth Points Against the Angle That Smooths the Drive* [Paper 557]. Zenodo. 10.5281/zenodo.23014224

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