A Skylight Is Twice as Bright as a Wall Window Only Under a Cloudy Sky

— Under a uniformly bright sky a horizontal skylight receives **2.000000** times the light of a wall window of the same size, and under an overcast sky whose zenith is three times brighter than its horizon, **2.524124** times — the winner changes at a solar altitude of **45** degrees — [Paper 587]

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— *From thinking to syncing.* —

Abstract

A skylight is said to be brighter than a wall window of the same size.What this paper shows is that the “twice” belongs to a cloudy day, when light comes from the whole sky, and that in direct sunlight the winner changes at a solar altitude of 45 degrees.No new theorem or law is claimed. First, under a uniformly bright sky, a skylight receives exactly 2.000000 times as much as a wall window——the light reaching a window is the sky’s luminance times the extent of sky the window sees weighted by its slant (the projected solid angle), which is π for a horizontal window and $\pi/2$ for a vertical one (Section 2). **Second, under the CIE standard overcast sky it is 2.524124 times ——the zenith is 3 times brighter than the horizon, and the skylight faces that bright zenith** (Section 2). **Third, and this is the core.In direct sunlight the ratio of wall window to skylight is set by the cotangent of the sun’s altitude h, $\cot h$, and the two are equal at 45 degrees——when the sun is lower than 45 degrees, the wall window facing it wins** (Section 3). **Fourth, compared at noon at latitude 35.7 degrees north, the winner changes with the season——at the winter solstice (altitude 30.86 degrees) the south wall window receives 1.673529 times the skylight; at the equinoxes (54.30 degrees) the skylight receives 1.391647 times the wall window, and at the summer solstice (77.74 degrees) 4.601847 times** (Section 3). **Fifth, the noon altitude falls below 45 degrees when the sun’s declination is south of -9.30 degrees——throughout that time, the wall window wins on a clear noon** (Section 3). **Sixth, the separator is whether light comes from the whole sky or from one direction——the same cosine projection makes the skylight win on a cloudy day and the wall window win on a clear winter noon** (Section 4). **Keywords**: daylighting, skylight, projected solid angle, CIE overcast sky, solar altitude.

Scope of this paper (scope note): **No new theorem or law is claimed——the cosine law for light reaching a surface, the luminance distributions of the uniform sky and of the CIE standard overcast sky, and the sun's noon altitude are all standard.No room is designed——window transmittance, reflection inside the room, window reveals and eaves, and surrounding buildings are not included.Only the light reaching the window plane is compared——the illuminance on a desk inside the room (the daylight factor) is not computed.Light reflected from the ground is not included——the wall window is taken to receive light from the sky alone; ground reflection would raise its value.Direct sunlight is compared as direct light only——the diffuse light of a clear sky is not included; the sun is at noon and the wall window faces due south.No dates are given——seasons are named by the sun's declination (± 23.44 and 0 degrees).Relation to earlier papers**: **Paper 214 showed that a lens can brighten an image but that passive optics cannot increase luminance ——a window does not increase luminance either; the brightness of its plane is set by the sky's luminance and by how much sky it sees, and at what slant.Paper 300 showed that sharing a root is decidable——the cloudy-sky "skylight 2 to 2.524124 times" and the direct-light "$\cot h$" both come from one root, the cosine projection onto the window plane.**What is added is **computing the illuminance of skylight and wall window under the uniform and overcast skies by two routes, numerical integration and closed forms**, **showing that in direct sunlight the winner changes at 45 degrees**, **giving the ratio at three noons at latitude 35.7 degrees north**, and **placing the separator on how widely the light's directions are spread**.

1. Notation and the Question

Setting: **let L be the luminance of the sky in the direction of altitude γ and azimuth ϕ.The illuminance reaching a horizontal skylight and a vertical wall window facing due south from the sky is**

$$E_{\mathrm{h}} = \int L \sin\gamma \, d\Omega, \qquad E_{\mathrm{v}} = \int L \cos\gamma \cos\phi \, d\Omega$$

(the wall window sees only the half of the sky in front of it, with ϕ measured from the direction it faces).

Observation 1.1

a skylight is said to be brighter than a wall window of the same size, twice as bright.A window facing up sees much of the sky——whether that makes it twice as bright always, or depending on the sky, is another question.

Question: **does a skylight always receive twice the light of a wall window?**

2. A Cloudy Sky

Computation 2.1: **the illuminance of the two windows under a uniform sky (luminance 1) and under the CIE standard overcast sky (zenith luminance 1, luminance $L = (1 + 2\sin\gamma)/3$), by numerical integration and in closed form.**

sky	skylight	wall window	skylight/wall window
uniform	**3.141592654** (π)	**1.570796327** ($\pi/2$)	**2.000000**
CIE overcast	**2.443460953** ($7\pi/9$)	**0.968043220**	**2.524124**

The closed form for the wall window under the overcast sky is $(2/3)(\pi/4 + 2/3)$, and numerical integration agrees with the closed forms to 6.6×10^{-11} or better for all four illuminances.

> **Observation 2.2**
>
> **the factor 2 under a uniform sky is pure geometry.A horizontal window sees the whole hemisphere of sky, weighted by the cosine of the slant; that extent is π. A vertical window sees only the half in front, and sees the overhead sky at a grazing angle, giving $\pi/2$——a window does not increase the sky's luminance; the light it receives is set by the extent of sky it sees.**

> **Observation 2.3**
>
> **under an overcast sky the skylight wins by more.In the CIE standard overcast sky the zenith is 3.000000 times brighter than the horizon. The skylight faces the bright zenith; the wall window faces the dim horizon ——the ratio rises from 2 to 2.524124, and the wall window receives only 0.396177 times the skylight.**

3. Direct Sunlight

Computation 3.1: **with the sun at altitude h and the wall window facing its azimuth, direct illuminance is proportional to $\sin h$ on the skylight and $\cos h$ on the wall window.The ratio is $\cot h$. At noon at latitude 35.7 degrees north,**

day	noon altitude	wall window/skylight	skylight/wall window
winter solstice	**30.86** degrees	**1.673529**	0.597540
equinox	**54.30** degrees	**0.718573**	**1.391647**
summer solstice	**77.74** degrees	**0.217304**	**4.601847**

Observation 3.2

this is the core.In direct sunlight the winner changes at a solar altitude of 45 degrees: at 44.9 degrees the wall window wins, at 45.1 degrees the skylight——at noon on the winter solstice the south wall window receives 1.673529 times the skylight. “A skylight is twice as bright” is reversed on a clear winter noon.

Observation 3.3

at latitude 35.7 degrees north the noon altitude falls below 45 degrees when the sun’s declination is south of −9.30 degrees.Throughout that time the south wall window wins on a clear noon——at the summer solstice, conversely, the skylight receives 4.601847 times the direct light of the wall window.

4. The Separator

Observation 4.1

the separator is whether light comes from the whole sky or from one direction.From the whole sky, what counts is how much sky the window sees, and the skylight wins by 2 to 2.524124 times. From one direction, what counts is only the angle between that direction and the window, and the winner changes at 45 degrees ——the same window wins or loses with the state of the sky.

Observation 4.2

beside Paper 214.It showed that a lens can brighten an image but that passive optics cannot increase luminance——a window is the same: it does not increase luminance, and all it can change is how much sky it sees, and at what slant.

Observation 4.3

one root, under Paper 300.The cloudy-sky ratio and the direct-light ratio both come from the cosine projection onto the window plane——what differs is only how widely the light's directions are spread.

Observation 4.4

what is not said.No room is designed, and window transmittance, reflection inside the room, surrounding buildings and ground reflection are not included——what is shown is that the ratio of light reaching the window planes from the sky is set by how widely the light's directions are spread, and changes at 45 degrees in direct sunlight.

5. Roll Call and Closing

1. **under a uniform sky the skylight receives $\mathbf{2.000000}$ times** (Computation 2.1): **π and $\pi/2$.**
2. **the factor 2 is pure geometry** (Observation 2.2): **the extent of sky seen.**
3. **$\mathbf{2.524124}$ times under the CIE overcast sky** (Computation 2.1): **the zenith $\mathbf{3.000000}$ times brighter than the horizon.**
4. **numerical integration and closed forms agree to 6.6×10^{-11}** (Computation 2.1): **two routes.**
5. **in direct sunlight the ratio is $\cot h$** (Computation 3.1): **equal at 45 degrees.**
6. **the wall window receives $\mathbf{1.673529}$ times at the winter noon** (Observation 3.2): **the core.**
7. **the skylight $\mathbf{1.391647}$ times at the equinox** (Computation 3.1): **altitude 54.30 degrees.**
8. **the skylight $\mathbf{4.601847}$ times at the summer solstice** (Observation 3.3): **altitude 77.74 degrees.**
9. **south of declination -9.30 degrees the wall window wins** (Observation 3.3): **on a clear noon.**
10. **the separator is how widely the light's directions are spread** (Observation 4.1): **whole sky or one direction.**

Closing: **A skylight is said to be brighter than a wall window of the same size, twice as bright.That belongs to a cloudy sky——under a uniformly bright sky a horizontal skylight receives exactly $\mathbf{2.000000}$ times as much as a wall window, because the extent of sky seen is π for a horizontal window and $\pi/2$ for a vertical one, and a window does not increase luminance; under the CIE standard overcast sky, whose zenith is 3 times brighter than its horizon, the factor is $\mathbf{2.524124}$.In direct sunlight the ratio is set by the cotangent of the sun's altitude and changes at 45 degrees——at noon on the winter solstice at latitude 35.7 degrees north (altitude 30.86 degrees) the south wall window receives $\mathbf{1.673529}$ times the skylight, while at the equinoxes the skylight receives $\mathbf{1.391647}$ times and at**

the summer solstice 4.601847 times. While the sun's declination is south of −9.30 degrees, the wall window wins on a clear noon.The separator is whether light comes from the whole sky or from one direction——the same cosine projection makes the skylight win on a cloudy day and the wall window win on a clear winter noon; under Paper 300 the two share a root.Placed among the earlier papers——as Paper 214 showed, luminance cannot be increased, so all a window can change is how much sky it sees, and at what slant.To be honest——only the light reaching the window planes directly from the sky was compared, without window transmittance, reflection inside the room, surrounding buildings, ground reflection or the diffuse light of a clear sky. The brightness inside a room was not computed, and no room is designed.

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