A Soap Film Turns Black Not Because the Film Is Gone

— a quarter-wave water film strengthens reflection to **3.844228** times a single water surface, while the same quarter wave on glass cuts it to **0.295932** — the difference is that the phase flips at only one surface, so a soap film grows darker as it thins — [Paper 588]

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— *From thinking to syncing.* —

Abstract

A soap bubble turns black just before it bursts. It is easily taken that the film has grown too thin to return light.What this paper shows is that a black film does not fail to return light: the light returned by its two surfaces cancels, because the phase flips at only one surface, and so the same quarter-wave thickness that kills reflection in an anti-reflection coating maximises it in a soap film.No new theorem or law is claimed. First, reflection is greatest at a quarter wave——one surface of a water film returns 0.020059 of the light; when the film is a quarter wave $\lambda/(4n)$ thick, the two reflections align to give 0.077113, 3.844228 times one surface (Section 2). **Second, dropping the phase flip confuses bright and dark——the first bright thickness is 75.1880 nm for violet (400 nm), 103.3835 nm for green (550 nm) and 131.5789 nm for red (700 nm). Counting no flip and taking $\lambda/(2n)$ gives 206.7669 nm as the bright thickness for green, but at that thickness green reflection vanishes exactly** (Section 2). **Third, and this is the core.As the film thins, reflection goes to 0——at 5 nm green reflection is 0.000481, and at zero thickness exactly 0. A black film does not fail to return light; the returned light cancels. As far as reflection can tell, a black film and a hole look the same** (Section 3). **Fourth, the thickness at which it turns dark depends on colour——the film becomes darker than a single water surface below 24.7845 nm for violet, 34.0787 nm for green and 43.3729 nm for red; a film 30 nm thick is still brighter than one water surface in violet (0.027940)** (Section 3). **Fifth, the same quarter wave weakens reflection in an anti-reflection coating——in the MgF_2 coating on glass of Paper 370, reflection falls from the bare glass's 0.042580 to 0.012601, 0.295932 times; at zero thickness it returns to the bare glass and does not go dark** (Section 4). **Sixth, the separator is whether the film's index lies between those of the media on either side or outside them——that is, the sign of $(n_1 - n_0)(n_1 - n_s)$: positive for the soap film (a flip at one surface only), negative for the coating (a flip at both)** (Section 5). **Keywords**: soap film, thin-film interference, black film, anti-reflection coating, phase reversal.

Scope of this paper (scope note): **No new theorem or law is claimed——thin-film interference, Fresnel reflection and the characteristic matrix of a single layer are all standard.Only normal incidence is computed——the shift of colour when the film is viewed obliquely is not treated.Absorption is not included, and refractive indices do not depend on wavelength——the water film is 1.33, and as in Paper 370 MgF_2 is 1.38 and glass 1.52.Why the film thins is not treated——drainage of liquid in the film, evaporation, flow driven by differences of surface tension and the time of bursting are not the subject.The appearance of colour is not computed——reflectance is given at three wavelengths and not converted into visible colour.Relation to earlier papers**: **Paper 370 showed that a single anti-reflection layer falls short for two independent reasons——this paper looks at the same thin-film interference with the order of indices reversed, and reproduces 370's quarter-wave reflectance 0.012601 and bare-glass 0.042580 by a separate route, the characteristic matrix.Paper 300 showed that sharing a root is decidable——the anti-reflection coating and the soap film come from the same formula and share a root.**What is added is **computing reflectance by two routes, the Airy formula and the characteristic matrix (differing by 4.2×10^{-17} or less), showing that green vanishes at the thickness obtained by dropping the phase flip, giving at three wavelengths the thickness below which the film is darker than one water surface**, and **placing the separator on the order of refractive indices**.

1. Notation and the Question

Setting: **a film of index n has air on both sides, and light falls on it normally.With the amplitude reflection of one surface $r = (n-1)/(n+1)$ and the phase delay of light crossing the film and back $\delta = 4\pi nt/\lambda$ (t the thickness), the reflectance of the whole film is**

$$R = \frac{2r^2(1-\cos\delta)}{1+r^4-2r^2\cos\delta}$$

(the Airy formula).Reflection flips the phase at the surface where light enters the film from air, and not at the surface where it leaves the film for air.

> **Observation 1.1**
>
> **a soap bubble turns black just before it bursts.It is easily taken that the film has grown too thin to return light——whether the film returns no light or the returned light cancels is another question.**

Question: **why does a soap film turn black, and what does the same thin-film interference do in an anti-reflection coating?**

2. Greatest at a Quarter Wave

Computation 2.1: one surface of a water film (index 1.33) reflects $r^2 = \mathbf{0.020059}$.When the film is $t = \lambda/(4n)$ thick, $\cos\delta = -1$ and the reflectance is at its greatest, $4r^2/(1+r^2)^2 = \mathbf{0.077113}$, $\mathbf{3.844228}$ times one surface.Computing by a separate route, the characteristic matrix of a single layer, gives

wavelength	first bright thickness	reflectance (Airy)	reflectance (matrix)
400 nm	**75.1880** nm	0.077113	0.077113
450 nm	84.5865 nm	0.077113	0.077113
550 nm	**103.3835** nm	0.077113	0.077113
700 nm	**131.5789** nm	0.077113	0.077113

and the two routes differ by 4.2×10^{-17} or less over some dozen combinations of thickness and wavelength.

> **Observation 2.2**
>
> reflection is greatest at a quarter wave because the phase flips at one surface only.Light that crosses the film and back is delayed by half a wave, and the flip at the entering surface adds another half, so the two reflections align——that the flip is at one surface only is what sets the direction of brightness.

> **Observation 2.3**
>
> dropping the phase flip confuses bright and dark.Counting no flip and saying “a round trip of one wave reinforces” gives $\lambda/(2n) = \mathbf{206.7669}$ nm as the first bright thickness for green——in fact at that thickness $\cos\delta = 1$ and green reflection vanishes exactly (1.3×10^{-33} by the matrix).

3. The Thin Limit

Computation 3.1: reflectance as the film thins.

thickness	400 nm (violet)	550 nm (green)	700 nm (red)
30 nm	**0.027940**	0.015933	0.010163
20 nm	0.013571	0.007426	0.004652
10 nm	0.003581	0.001910	0.001184
5 nm	0.000908	**0.000481**	0.000297

> **Observation 3.2**
>
> this is the core.As the film thins, reflection goes to 0, and at zero thickness it is exactly 0——when the two surfaces coincide, the flipped reflection and the unflipped one cancel exactly. A black film does not fail to return light; the returned light cancels. As far as reflection can tell, a black film and a hole look the same.

Computation 3.3: the film is darker than a single water surface, $R < r^2$, when $\cos\delta > (1 + r^2)/2$, that is, below **24.7845** nm for violet, **34.0787** nm for green and **43.3729** nm for red.At the green boundary the reflectance matches r^2 to 3.5×10^{-18}.

> **Observation 3.4**
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> the thickness at which the film turns dark depends on colour.A film 30 nm thick is darker than one water surface in green and red but still brighter in violet (0.027940)——only below 24.7845 nm is it darker than one water surface in every colour.

4. Beside the Anti-Reflection Coating

Computation 4.1: the anti-reflection coating of Paper 370, MgF_2 (index 1.38) on glass (1.52) at a quarter-wave thickness (wavelength 550 nm).The bare glass reflects **0.042580**; with the coating, **0.012601** (matching the closed form $((n_s - n_1^2)/(n_s + n_1^2))^2$), or **0.295932** times the bare glass.At zero coating thickness it returns to 0.042580.

Observation 4.2

here the same quarter wave weakens reflection.From air into MgF_2 and from MgF_2 into glass, light enters the higher index both times, so the phase flips at both surfaces——with two flips, only the half-wave delay of the round trip remains and the two reflections cancel; and at zero thickness the glass is simply bare again, not dark.

5. The Separator

Observation 5.1

the separator is the sign of $(n_1 - n_0)(n_1 - n_s)$.The soap film gives $+0.1089$ (its index above both sides, a flip at one surface only); the coating gives -0.0532 (its index between the two sides, a flip at both)——if positive, reflection is greatest at a quarter wave and vanishes at zero thickness; if negative, it is least at a quarter wave and returns to the bare surface at zero thickness.

Observation 5.2

beside Paper 370.It gave two reasons why a single anti-reflection layer falls short——this paper looks at the other side of the same formula, with the order of indices reversed. One quarter wave becomes a cancelling thickness or a reinforcing one according to that order alone.

Observation 5.3

one root, under Paper 300.The coating and the soap film come from the same Airy formula and the same characteristic matrix——what differs is only the order of indices.

Observation 5.4

what is not said.Why the film thins and when it bursts are not treated, nor is the appearance of colour computed——what is shown is only that a black film cancels its reflection, and that the order of indices decides this.

6. Roll Call and Closing

1. **one water surface reflects 0.020059** (Computation 2.1): **0.077113 at a quarter wave.**
2. **3.844228 times one surface** (Computation 2.1): **the two reflections align.**
3. **the first bright thickness for green is 103.3835 nm** (Computation 2.1): **violet 75.1880, red 131.5789.**
4. **dropping the flip mistakes 206.7669 nm** (Observation 2.3): **where green vanishes.**
5. **thinning sends reflection to 0** (Observation 3.2): **the core.**
6. **green reflects 0.000481 at 5 nm** (Computation 3.1): **a black film and a hole look the same.**
7. **darker than one surface below 24.7845 nm in violet** (Computation 3.3): **green 34.0787, red 43.3729.**
8. **the coating takes 0.042580 to 0.012601** (Computation 4.1): **the same quarter wave weakens.**
9. **the two routes differ by 4.2×10^{-17} or less** (Computation 2.1): **Airy formula and matrix.**
10. **the separator is the sign of $(n_1 - n_0)(n_1 - n_s)$** (Observation 5.1): **the order of indices.**

Closing: **A soap bubble turns black just before it bursts, and it is easily taken that the film has grown too thin to return light.A black film does not fail to return light; the returned light cancels——one surface of a water film returns 0.020059 of the light, and the phase flips only where light enters the film from air. So at a quarter-wave thickness (103.3835 nm for green) the two reflections align to give 0.077113, 3.844228 times one surface, and as the thickness goes to 0 they cancel and reflection goes to 0: green reflects 0.000481 at 5 nm, and only below 24.7845 nm is the film darker than one water surface in every colour. Dropping the phase flip mistakes 206.7669 nm, where green vanishes, for the first bright thickness.The same quarter wave weakens reflection in an anti-reflection coating——in the MgF_2 coating of Paper 370 the phase flips at both surfaces, so reflection falls from the bare glass's 0.042580 to 0.012601, 0.295932 times, and at zero thickness the glass is bare again.The separator is whether the film's index lies between those on either side or outside them, the sign of $(n_1 - n_0)(n_1 - n_s)$——under Paper 300 the two come from the same formula and share a root, differing only in the order of indices.To be honest——only normal incidence was computed, without absorption and with indices independent of wavelength. Why the film thins and when it bursts are not treated, and the appearance of colour was not computed.**

References

1. Born, M., & Wolf, E. (1999). *Principles of Optics* (7th ed.). Cambridge University Press.
2. Hecht, E. (2017). *Optics* (5th ed.). Pearson.
3. Isenberg, C. (1992). *The Science of Soap Films and Soap Bubbles*. Dover.
4. Yamagishi, Y. (2026). *The Three Hundredth Census: Whether Two Things Share a Root Is Decidable* [Paper 300]. Zenodo. 10.5281/zenodo.22171914

5. Yamagishi, Y. (2026). *A Single Anti-Reflection Layer Falls Short for Two Independent Reasons* [Paper 370]. Zenodo. 10.5281/zenodo.22697804

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