A Clip on a Paper Aeroplane Does Not Shorten How Far It Glides

— adding a 1 g clip to a paper aeroplane of one A4 sheet makes it **1.095635** times faster and its time aloft **0.912712** times as long, but leaves the glide angle and the distance of **12.356432** m unchanged — to compete in time, fly at **0.747049** times the speed of greatest lift-to-drag ratio — [Paper 590]

Yuuki YAMAGISHI (山岸 勇輝)
Yamagishi Complexity Torus Lab, Japan
September 2026

— *From thinking to syncing.* —

Abstract

A heavy paper aeroplane is easily taken not to fly far.What this paper shows is that the glide angle is set by the ratio of lift to drag alone and does not depend on weight, that what weight changes is the speed and the time aloft, and that the speed at which to fly differs according to whether one competes in distance or in time.No new theorem or law is claimed. First, the glide angle is set by the lift coefficient alone——in a steady glide, the tangent of the glide angle is the ratio of drag to lift. With a representative drag law, the greatest lift-to-drag ratio is 6.864684, at a glide angle of 8.288157 degrees (Section 2). **Second, and this is the core.A 1 g clip that makes the aeroplane 1.200417 times heavier leaves the distance unchanged——the speed and the rate of sink become 1.095635 times (the square root of the weight ratio) and the time aloft falls from 3.996122 to 3.647310 seconds, 0.912712 times, but the distance from a height of 1.8 m stays at 12.356432 m** (Section 3). **Third, solving the equations of motion directly gives the same——integrating the equations of motion of a point mass numerically, distance and time agree with the steady formulas to within 4.2×10^{-13}** (Section 3). **Fourth, to compete in time, fly at the lift coefficient of least sink ——in the textbook closed forms, which take the angle to be small, that speed is $3^{-1/4} = 0.759836$ times the speed of best glide and the lift-to-drag ratio $\sqrt{3}/2 = 0.866025$ times** (Section 4). **Fifth, putting in the glide angle of 8 degrees correctly moves the numbers a little——optimising the exact formulas numerically, the speed is 0.747049 times, the lift-to-drag ratio (that is, the distance) 0.852819 times and the time aloft 1.146023 times** (Section 4). **Sixth, the separator is whether one competes in distance or in time——different points on the same drag law are best; weight moves neither point and changes only the speed, by a square root** (Section 5). **Keywords**: paper aeroplane, glide ratio, lift-to-drag ratio, minimum sink, gliding.

Scope of this paper (scope note): No new theorem or law is claimed——the balance of a steady glide, the drag law in which drag grows with the square of the lift coefficient (the parabolic polar), and best glide and least sink are all standard.The drag law is representative——a profile-drag coefficient of 0.04, aspect ratio 3, efficiency 0.8, wing area $0.015\ \mathrm{m}^2$ and air density $1.204\ \mathrm{kg/m}^3$.The coefficients are taken not to depend on speed——at the speeds of a paper aeroplane they do in fact change; that is not included.Stall is not included——whether the least-sink lift coefficient of 0.980229 lies below the stall was not checked.The oscillation after launch and the wind are not included——only the steady glide is considered.No paper aeroplane is designed.Relation to earlier papers: Paper 458 showed that raising the lift coefficient by sixty per cent lowers the stall speed by only twenty, since it acts through a square root——here too speed moves with the square root of weight; the same square root, but the distance does not depend on weight.Paper 390 showed that the lift-to-drag ratio decides the ratio of a sailing craft's speed to the wind's——here the lift-to-drag ratio decides the distance.Paper 300 showed that sharing a root is decidable——best glide and least sink come from the same drag law and share a root, differing only in what is optimised.What is added is confirming by two routes, the steady formulas and the equations of motion, that changing the weight leaves the glide angle and distance unchanged, giving in numbers the shortening of time aloft by the square root of the weight ratio, giving the least-sink speed both by the small-angle closed forms and by the exact formulas, and placing the separator on distance or time.

1. Notation and the Question

Setting: a paper aeroplane of mass m, wing area S and lift coefficient C_L descends steadily along a path at glide angle γ.From the balance of lift, drag and weight,

$$\tan\gamma = \frac{C_D}{C_L}, \qquad v = \sqrt{\frac{2mg\cos\gamma}{\rho S C_L}}, \qquad C_D = C_{D0} + K C_L^2,$$

with $K = 1/(\pi e A)$ (A the aspect ratio, e the efficiency).From a height H the distance is $H/\tan\gamma$ and the time $H/(v\sin\gamma)$.

Observation 1.1

a heavy paper aeroplane is easily taken not to fly far.With a clip on, it seems to fall faster——falling faster and landing nearer are different things.

Question: does weight change how far a paper aeroplane flies?

2. The Glide Angle

Computation 2.1: with a profile-drag coefficient $C_{D0} = 0.04$, aspect ratio 3 and efficiency 0.8, $K = 0.132629$.The lift coefficient of greatest lift-to-drag ratio is $C_L^* = \sqrt{C_{D0}/K} = 0.549175$ (the same when optimised numerically), giving a lift-to-drag ratio of **6.864684** and a glide angle of **8.288157** degrees.

> **Observation 2.2**
>
> weight does not enter the formula for the glide angle.$\tan\gamma = C_D/C_L$ is set by the lift coefficient alone—— weight enters only the formula for speed.

3. Changing the Weight

Computation 3.1: one sheet of A4 (80 g/m^2) weighs 4.9896 g; with a wing area of 0.015 m^2 and air density 1.204 kg/m^3, flown at the lift coefficient of greatest lift-to-drag ratio from a height of 1.8 m:

	glide angle	speed	sink	time aloft	distance
aeroplane alone (4.9896 g)	**8.288157** degrees	3.124742 m/s	0.450437 m/s	**3.996122** s	**12.356432** m
with a 1 g clip (5.9896 g)	**8.288157** degrees	3.423578 m/s	0.493514 m/s	**3.647310** s	**12.356432** m

> **Observation 3.2**
>
> this is the core.When the weight becomes 1.200417 times, the speed and the rate of sink become **1.095635** times (the square root of the weight ratio) and the time aloft **0.912712** times, but the distance does not change——the aeroplane with a clip descends the same path at the same angle, faster. It lands not nearer, but sooner.

Computation 3.3: as a route separate from the steady formulas, the equations of motion of a point mass under lift, drag and weight were integrated numerically (Runge--Kutta).Alone: distance 12.356432 m and time 3.996122 s; with the clip: 12.356432 m and 3.647310 s, agreeing with the steady formulas to within 4.2×10^{-13}.

4. Distance or Time

Computation 4.1: **the lift coefficient that minimises the rate of sink $v\sin\gamma$ is $\sqrt{3}C_L^* = 0.951199$ if the angle is taken to be small, and 0.980229 when the exact formulas are minimised numerically.Relative to best glide,**

ratio	small-angle closed form	exact formulas
speed	**0.759836** ($3^{-1/4}$)	**0.747049**
lift-to-drag (distance)	**0.866025** ($\sqrt{3}/2$)	**0.852819**
rate of sink	0.877383	0.872583

By the exact formulas, best glide gives 3.124742 m/s, 12.356432 m and 3.996122 s; least sink gives 2.334334 m/s, 10.537797 m and 4.579646 s.

> **Observation 4.2**
>
> **to compete in time, fly slower.The least-sink speed is 0.747049 times the best-glide speed, and the time aloft grows 1.146023 times, but the distance falls to 0.852819 times——the best for distance and the best for time lie at different speeds of the same aeroplane.**

> **Observation 4.3**
>
> **the textbook closed forms take the angle to be small.At a glide angle of 8 degrees the speed ratio is 0.747049 rather than 0.759836, and the lift-to-drag ratio 0.852819 rather than 0.866025——the direction is the same, but the numbers are found with the angle put in.**

5. The Separator

Observation 5.1

the separator is whether one competes in distance or in time.For distance, fly at the point of greatest lift-to-drag ratio; for time, at the point of least sink. They are different points on the same drag law——weight moves the lift coefficient of neither point and changes only the speed, by a square root. So weight changes the time and not the distance.

Observation 5.2

beside Paper 458.It showed that raising the lift coefficient by sixty per cent lowers the stall speed by only twenty, since it acts through a square root——here too speed follows weight through a square root; the same square root, but the distance lies outside it.

Observation 5.3

beside Paper 390.It showed that the lift-to-drag ratio decides the ratio of a sailing craft's speed to the wind's ——here the lift-to-drag ratio decides the ratio of distance to height.

Observation 5.4

one root, under Paper 300.Best glide and least sink come from the same drag law——they differ only in whether distance or time is optimised.

Observation 5.5

what is not said.No paper aeroplane is designed, and the change of coefficients with speed, stall, the oscillation after launch and the wind are not included——what is shown is that weight changes the time and not the distance, and that the best speeds for distance and for time differ.

6. Roll Call and Closing

1. **the greatest lift-to-drag ratio is 6.864684** (Computation 2.1): **a glide angle of 8.288157 degrees.**
2. **weight does not enter the glide angle** (Observation 2.2): **set by the lift coefficient alone.**
3. **the clip makes it 1.095635 times faster** (Observation 3.2): **the square root of the weight ratio.**
4. **time aloft from 3.996122 to 3.647310 seconds** (Computation 3.1): **0.912712 times.**
5. **the distance stays at 12.356432 m** (Observation 3.2): **the core.**
6. **the equations of motion agree to within 4.2×10^{-13}** (Computation 3.3): **two routes.**
7. **to compete in time, 0.747049 times the speed** (Observation 4.2): **time aloft 1.146023 times.**
8. **the distance then falls to 0.852819 times** (Observation 4.2): **distance and time at different speeds.**
9. **the closed forms give 0.759836 and 0.866025** (Observation 4.3): **values for a small angle.**
10. **the separator is distance or time** (Observation 5.1): **different points on the same drag law.**

Closing: **A heavy paper aeroplane is easily taken not to fly far.The glide angle is set by the lift coefficient alone and does not depend on weight——with a representative drag law the greatest lift-to-drag ratio is 6.864684 and the glide angle 8.288157 degrees. Adding a 1 g clip to a paper aeroplane of one sheet of A4 (4.9896 g) makes the speed and the rate of sink 1.095635 times and the time aloft fall from 3.996122 to 3.647310 seconds, 0.912712 times, but the distance from a height of 1.8 m stays at 12.356432 m, the same to within 4.2×10^{-13} when the equations of motion are solved. The aeroplane with a clip does not land nearer; it descends the same path sooner.The speed at which to fly differs between competing in distance and competing in time——to compete in time, fly at the point of least sink, at 0.747049 times the best-glide speed, which lengthens the time aloft 1.146023 times but cuts the distance to 0.852819 times. The textbook closed forms, taking the angle to be small, give 0.759836 and 0.866025: the same direction, but the numbers are found with the angle put in.The separator is whether one competes in distance or in time——different points on the same drag law are best, and under Paper 300 the two share a root.Placed among the earlier papers——speed follows weight through the same square root as in Paper 458, but the distance lies outside it; the lift-to-drag ratio that set the speed ratio of a sailing craft in Paper 390 here sets the distance.To be honest——the drag law and wing area are representative, and the change of coefficients with speed, stall, the oscillation after launch and the wind are not included. Whether the least-sink lift coefficient lies below the stall was not checked, and no paper aeroplane is designed.**

References

1. Anderson, J. D. (2016). *Introduction to Flight* (8th ed.). McGraw-Hill Education.
2. McCormick, B. W. (1995). *Aerodynamics, Aeronautics, and Flight Mechanics* (2nd ed.). Wiley.
3. Tennekes, H. (2009). *The Simple Science of Flight: From Insects to Jumbo Jets*. MIT Press.

4. Yamagishi, Y. (2026). *The Three Hundredth Census: Whether Two Things Share a Root Is Decidable* [Paper 300]. Zenodo. 10.5281/zenodo.22171914

5. Yamagishi, Y. (2026). *Going Faster Makes the Wind Stronger, Which Is Why a Sailing Craft Can Outrun the Wind* [Paper 390]. Zenodo. 10.5281/zenodo.22699203

6. Yamagishi, Y. (2026). *Raising the Lift Coefficient by Sixty Per Cent Lowers the Stall Speed by Twenty* [Paper 458]. Zenodo. 10.5281/zenodo.22733085

— From thinking to syncing. —

Yuuki YAMAGISHI (山岸 勇輝)

© 2026 Yamagishi Complexity Torus Lab, Japan · Licensed under Creative Commons Attribution 4.0 International (CC BY 4.0)