A Rubber Balloon Swells as It Rises, but Its Ascent Speeds Up Only as the −1/6 Power of the Air Density

— a rubber balloon swollen to **64.0000** times its ground volume is rising only exactly **2.000000** times as fast as at launch — a rigid balloon of fixed volume stops at **2650.3** m even with **30** per cent more lift than weight — [Paper 592]

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— From thinking to syncing. —

Abstract

A balloon swells as it rises, and it is easily taken that it floats less and less as the air thins.What this paper shows is that in a rubber balloon the amount of gas inside does not change, so the net lift is the same at every altitude; swelling preserves the lift, and the ascent speed changes only as the −1/6 power of the air density.No new theorem or law is claimed. First, the volume swells in inverse proportion to the air density——a balloon launched at 1.5 m in diameter has 13.9149 times its volume and 2.4053 times its diameter at 20000 m (Section 2). **Second, the net lift is constant——the product of air density and volume does not change, so the weight of displaced air less the weight of helium (0.861810 of the displaced air's weight) does not depend on altitude** (Section 2). **Third, and this is the core.The ascent speed goes as the −1/6 power of the air density——drag is proportional to the product of air density and cross-section, and the cross-section goes as the 2/3 power of volume, that is, as the −2/3 power of density, so the product falls only as the 1/3 power of density. The speed is 1.550887 times its launch value at 20000 m and exactly 2.000000 times at the burst altitude of 29608.6 m, the square root of the ratio of burst to launch diameter** (Section 3). **Fourth, the time to burst is shorter than a constant-speed estimate——73.40 minutes, 0.743667 times the 98.70 minutes of an ascent kept at the launch speed** (Section 3). **Fifth, a rigid balloon stops——with fixed volume the lift falls in proportion to density; a balloon with 10 per cent more lift than weight at launch comes to rest at 981.8 m, and one with 30 per cent at 2650.3 m** (Section 4). **Sixth, the separator is whether the balloon can swell——a balloon that can swell keeps its lift and rises until it bursts; one that cannot loses its lift and stops** (Section 5). **Keywords**: rubber balloon, ascent rate, buoyancy, standard atmosphere, drag.

Scope of this paper (scope note): No new theorem or law is claimed——the standard atmosphere, buoyancy and the drag law are all standard.The 1976 U.S. Standard Atmosphere is used——altitudes are geopotential, and only the three layers from 0 to 32 km are used.The gas inside is taken to be at the temperature and pressure of the surrounding air——solar heating and overpressure from the rubber's tension are not included.The drag coefficient is taken to be constant——the speed ratio does not depend on its value, but its change with Reynolds number is not included.The launch diameter of 1.5 m, burst diameter of 6 m and launch ascent speed of 5 m/s are representative.Crosswinds, the elastic properties of the rubber and the mechanism of bursting are not treated.No balloon is designed.**Relation to earlier papers**: Paper 326 showed that the square-cube relation is not a law but an identity——here the identity that area goes as the $2/3$ power of volume sets the exponent $-1/6$ of the ascent speed.Paper 300 showed that sharing a root is decidable——the swelling balloon and the rigid balloon come from the same buoyancy equation and share a root; only whether the volume can change separates them.What is added is giving volume, diameter and ascent speed at each altitude in the standard atmosphere, confirming the $-1/6$ power by two routes, force balance and closed form, showing that the speed ratio at burst is the square root of the diameter ratio, and setting the rigid balloon beside it.

1. Notation and the Question

Setting: the amount of gas in a rubber balloon does not change, and its temperature and pressure are those of the surrounding air.Then the volume is inversely proportional to the air density, and the weight of displaced air does not change.With launch density and volume ρ_0, V_0, balloon and payload weight mg, drag coefficient C_D and cross-section A,

$$\rho V = \rho_0 V_0, \qquad F = \Big(1 - \frac{M_{\mathrm{He}}}{M_{\mathrm{air}}}\Big)\rho_0 V_0\, g - mg, \qquad \frac{1}{2}\rho v^2 C_D A = F,$$

and the cross-section is proportional to the $2/3$ power of the volume.

> **Observation 1.1**
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> a balloon swells as it rises, and it is easily taken that it floats less and less as the air thins.That thinner air means less lift is right for rigid bodies——whether it is right for bodies that swell is another question.

Question: how do the lift and the ascent speed of a rubber balloon change as it rises?

2. A Swelling Balloon

Computation 2.1: in the 1976 U.S. Standard Atmosphere the ground density is 1.224999 kg/m^3; at 11000 m the air is at 216.65 K, 22632.06 Pa and 0.363918 kg/m^3, at 20000 m at 5474.89 Pa and 0.088035 kg/m^3, and at 30000 m at

226.65 K, 1171.87 Pa and 0.018012 kg/m^3.A balloon launched at 1.5 m in diameter and 5 m/s, bursting at 6 m in diameter, gives

altitude	volume (times ground)	diameter (times)	ascent speed
0 m	1.0000	1.0000	5.0000 m/s
5000 m	1.6641	1.1850	5.4430 m/s
10000 m	2.9682	1.4371	5.9940 m/s
15000 m	6.3251	1.8494	6.7996 m/s
20000 m	**13.9149**	**2.4053**	7.7544 m/s
25000 m	31.0395	3.1427	8.8639 m/s
29608.6 m (burst)	**64.0000**	4.0000	10.0000 m/s

The density at burst is 0.019141 kg/m^3, one sixty-fourth of the ground value.

> **Observation 2.2**
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> the lift does not depend on altitude.Since the product of air density and volume is constant, the weight of displaced air is constant, and so is the weight of the helium inside. Buoyancy less the helium's weight is **0.861810** of the displaced air's weight, whatever the altitude——the swelling exactly makes up for the thinning air.

3. The Ascent Speed

Computation 3.1: with constant lift, the ascent speed is set by the balance with drag, $v \propto (\rho A)^{-1/2} \propto \rho^{-1/6}$.The speed obtained directly from the force balance and this closed form differ by a relative 2.2×10^{-16}.At the burst altitude the density is one sixty-fourth of the ground value, so the speed is exactly $64^{1/6} = \mathbf{2.000000}$ times the launch speed, the square root of the ratio 4 of burst to launch diameter.The time to burst is 4403.8 seconds = 73.40 minutes, **0.743667** times the 98.70 minutes of an ascent kept at the launch 5 m/s.

Observation 3.2

this is the core.The volume swells 64-fold, but the ascent speed only doubles——drag is proportional to the product of air density and cross-section. The density falls to one sixty-fourth, but the cross-section, going as the $2/3$ power of volume, grows 16-fold, so the product falls only to a quarter, and the speed, as its inverse square root, doubles.

4. A Rigid Balloon

Computation 4.1: in a rigid balloon of fixed volume, the lift falls in proportion to the air density.A balloon with 10 per cent more lift than weight at launch comes to rest at 981.8 m, where the density is 0.909091 of the ground value; one with 30 per cent comes to rest at 2650.3 m, at 0.769231.

Observation 4.2

the same thinning air stops the rigid balloon and not the rubber one.The rigid balloon loses lift as the air thins——the rubber balloon swells as the air thins and loses none. A rubber balloon stops not when its lift runs out but when the rubber, stretched to its limit, bursts.

5. The Separator

Observation 5.1

the separator is whether the balloon can swell.A balloon that can swell keeps its lift and rises until it bursts, its speed growing slowly as the $-1/6$ power of density. One that cannot loses its lift and stops where lift and weight balance——“thinner air, less floating” is right only for bodies that cannot swell.

Observation 5.2

the identity of Paper 326 sets the exponent.It showed that the square-cube relation is not a law but an identity——here the cross-section as the $2/3$ power of volume and the volume as the -1 power of density combine to give the exponent $-1/6$ of the ascent speed.

Observation 5.3

one root, under Paper 300.The swelling balloon and the rigid balloon come from the same buoyancy equation——what separates them is only whether the volume can change.

Observation 5.4

what is not said.No balloon is designed; solar heating, overpressure, changes in the drag coefficient and crosswinds are not included——what is shown is the form of the equations: swelling preserves lift, and speed changes only as the $-1/6$ power of density.

6. Roll Call and Closing

1. **at 20000 m, volume 13.9149 times and diameter 2.4053 times** (Computation 2.1): **swelling as the inverse of density.**
2. **net lift constant at 0.861810 of displaced air** (Observation 2.2): **independent of altitude.**
3. **ascent speed as the $-1/6$ power of density** (Computation 3.1): **agreeing with the force balance to 2.2×10^{-16}.**
4. **1.550887 times at 20000 m** (Computation 2.1): **far less than the swelling.**
5. **volume 64.0000 times and speed 2.000000 times at burst** (Observation 3.2): **the core.**
6. **the speed ratio is the square root of the diameter ratio** (Computation 3.1): $\sqrt{4} = 2$**.**
7. **73.40 minutes to burst** (Computation 3.1): **0.743667 times the constant-speed estimate.**
8. **rigid balloons stop at 981.8 m and 2650.3 m** (Computation 4.1): **lift falls with density.**
9. **the identity of Paper 326 sets the exponent** (Observation 5.2): **area as the $2/3$ power of volume.**
10. **the separator is whether the balloon can swell** (Observation 5.1): **keeping lift or losing it.**

Closing: **A balloon swells as it rises, and it is easily taken that it floats less and less as the air thins.In a rubber balloon the net lift is the same at every altitude——the amount of gas inside does not change, so the product of air density and volume does not change, and buoyancy less the helium's weight stays at 0.861810 of the displaced air's weight. A balloon launched at 1.5 m swells to 13.9149 times its volume and 2.4053 times its diameter at 20000 m.The ascent speed changes only as the $-1/6$ power of the air density——the cross-section widens as the $2/3$ power of volume, so the product of density and cross-section that sets the drag falls only as the $1/3$ power of density. At the burst altitude of 29608.6 m (geopotential), where a 6 m balloon bursts, the volume is 64.0000 times but the speed exactly 2.000000 times the launch value, the square root of the diameter ratio, and the 73.40 minutes to burst are**

0.743667 times a constant-speed estimate. A rigid balloon of fixed volume stops at 981.8 m with 10 per cent more lift than weight, and at 2650.3 m with 30 per cent.The separator is whether the balloon can swell——"thinner air, less floating" is right only for bodies that cannot swell; under Paper 300 the two come from one buoyancy equation and share a root.Placed among the earlier papers——the identity of Paper 326, area as the $2/3$ power of volume, here sets the speed exponent $-1/6$.To be honest——the gas inside is taken to be at the air's temperature and pressure, and solar heating, overpressure, changes in the drag coefficient and crosswinds are not included. The diameters and launch speed are representative, and no balloon is designed.

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