Braking at Half Does Not Halve the Grip for Turning: Tyre Friction Shares One Circle Between Braking and Turning

— With a tyre of friction coefficient 0.8, using half the friction for braking still leaves $\mathbf{0.866025}$ for turning, and using nine-tenths leaves only $\mathbf{0.435890}$ — as the sideways friction approaches its limit, this ratio approaches $\pi/2 = \mathbf{1.570796}$ — [Paper 593]

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— From thinking to syncing. —

Abstract

Braking at half is easily taken to halve the margin for turning.What this paper shows is that tyre friction is shared between braking and turning within one circle, so a little braking barely reduces the turning force while hard braking reduces it greatly, and that stopping while turning takes $\sin^{-1} y/y$ times the distance of a straight stop (y being the fraction of friction used sideways).No new theorem or law is claimed. First, the fraction of friction left for turning is $\sqrt{1-b^2}$——using a fraction b for braking leaves $\mathbf{0.866025}$ at $b = 0.5$ (addition would say 0.5) and $\mathbf{0.435890}$ at $b = 0.9$ (Section 2). **Second, the first braking barely touches the turning grip——using 30 per cent of the friction for braking reduces the turning grip by only $\mathbf{4.6061}$ per cent; using 90 per cent reduces it by $\mathbf{56.4110}$ per cent** (Section 2). **Third, the smallest radius one can turn grows $\mathbf{1.154701}$-fold with half braking and $\mathbf{2.294157}$-fold with nine-tenths——at 60 km/h with friction coefficient 0.8, 35.4068 m becomes 40.8843 m and 81.2288 m** (Section 3). **Fourth, and this is the core.Stopping while turning takes $\sin^{-1} y/y$ times the distance of a straight stop ——taking a 50 m radius curve at 60 km/h and braking with all the friction left over from turning, the car stops in $\mathbf{19.6714}$ m, $\mathbf{1.111163}$ times the $\mathbf{17.7034}$ m of a straight stop; the closed form and numerical integration agree** (Section 4). **Fifth, near the limit the ratio approaches $\pi/2$——for sideways use y of 0.5, 0.9 and 0.99 the ratio is $\mathbf{1.047198}$, $\mathbf{1.244188}$ and $\mathbf{1.443694}$, tending to $\pi/2 = \mathbf{1.570796}$ as $y \to 1$; exactly at the limit, all the friction is used sideways and braking cannot begin** (Section 4). **Sixth, the separator is whether braking and turning friction are treated as adding like numbers or as a quantity with direction——the friction limit applies to the combined length** (Section 5). **Keywords**: friction circle, tyre, braking, cornering, vehicle dynamics.

Scope of this paper (scope note): **No new theorem or law is claimed——the friction circle, centripetal acceleration and stopping distance are all standard.The car is a point mass and the friction circle is an idealisation——a real tyre has different limits along and across (an ellipse), varying with slip angle; here it is a circle.A friction coefficient of 0.8 is representative of a dry road.Load transfer (weight moving to the front wheels under braking), anti-lock systems, tyre elasticity and road banking are not included.No driving advice is given——this paper does not recommend any particular way of driving.Relation to earlier papers**: **Paper 453 treated a banked curve on an icy road as a point mass and found the band of safe speeds——it used only the sideways friction. This paper adds friction along the road (braking) to the same point mass and shows that the limit applies to the combined length.Paper 300 showed that sharing a root is decidable——stopping on a straight and stopping on a curve come from the same friction circle and share a root; only the sideways use y separates them ($y \to 0$ returns the straight).**What is added is **setting the fraction left for turning beside the additive assumption**, **deriving the closed form $(R/2)\sin^{-1}(v^2/(R\mu g))$ for stopping while turning and checking it against numerical integration**, **showing that the ratio $\sin^{-1} y/y$ tends to $\pi/2$ at the limit**, and **placing the separator on addition versus a quantity with direction**.

1. Notation and the Question

Setting: **treat the car as a point mass and split its acceleration along the road into a longitudinal part (braking) a_x and a lateral part (turning) a_y.The limit of acceleration the tyres can supply, μg, applies to the combined length:**

$$\sqrt{a_x^2 + a_y^2} \le \mu g.$$

Using a fraction $b = a_x/(\mu g)$ for braking leaves $\mu g\sqrt{1 - b^2}$ for turning.

> **Observation 1.1**
>
> **braking at half is easily taken to halve the margin for turning.Friction is seen as one purse paid out to braking and to turning——whether the purse empties by addition or in some other way is another question.**

Question: **how much does using friction for braking reduce the turning force, and how far does it take to stop while turning?**

2. Circle and Addition

Computation 2.1: **the fraction b used for braking and the fraction left for turning.**

b	circle $\sqrt{1-b^2}$	loss of turning grip	additive assumption $1-b$
0.10	0.994987	0.5013 per cent	0.900000
0.30	0.953939	**4.6061** per cent	0.700000
0.50	**0.866025**	13.3975 per cent	0.500000
0.70	0.714143	28.5857 per cent	0.300000
0.90	**0.435890**	**56.4110** per cent	0.100000
0.99	0.141067	85.8933 per cent	0.010000

> **Observation 2.2**
>
> the first braking barely touches the turning grip.At the top of the circle the fraction left falls only as the square of b: using 30 per cent for braking costs only 4.6061 per cent of the turning grip——the last tenth is the dearest; at nine-tenths, more than half the turning grip is gone.

3. The Radius One Can Turn

Computation 3.1: at 60 km/h with friction coefficient 0.8, the smallest radius $v^2/(\mu g\sqrt{1-b^2})$ is 35.4068 m without braking, 40.8843 m (**1.154701** times) with half the friction used for braking, and 81.2288 m (**2.294157** times) with nine-tenths.

> **Observation 3.2**
>
> with half braking the radius widens by only 15 per cent.Addition would double the radius; the circle gives 1.154701——but at nine-tenths braking it is 2.294157, and almost no margin for turning remains.

4. Stopping While Turning

Computation 4.1: a car on a curve of radius R at speed v_0 brakes with all the friction left over from turning.At speed v turning needs v^2/R, leaving a deceleration $\sqrt{(\mu g)^2 - v^4/R^2}$, which grows as the car slows.The stopping distance is

$$s=\int_0^{v_0}\frac{v\,dv}{\sqrt{(\mu g)^2-v^4/R^2}}=\frac{R}{2}\sin^{-1}\frac{v_0^2}{R\mu g},\qquad \frac{s}{v_0^2/(2\mu g)}=\frac{\sin^{-1}y}{y},\quad y=\frac{v_0^2}{R\mu g},$$

with y the fraction of friction used sideways.

sideways use y	speed	straight stop	stop on curve	ratio $\sin^{-1}y/y$
0.5	50.4171 km/h	12.5000 m	13.0900 m	**1.047198**
0.708136	60 km/h	**17.7034** m	**19.6714** m	**1.111163**
0.9	67.6416 km/h	22.5000 m	27.9942 m	**1.244188**
0.99	70.9431 km/h	24.7500 m	35.7314 m	**1.443694**

The radius is 50 m. Numerical integration and the closed form agree to 5.0×10^{-12} or better in all four.

Observation 4.2

this is the core.Stopping while taking a 50 m curve at 60 km/h takes 19.6714 m, 1.111163 times a straight stop——turning uses 0.708136 of the friction sideways, so at first only $\sqrt{1-y^2}=0.706076$ is left for braking; as the car slows, turning needs less and more is left for braking.

Observation 4.3

near the limit the ratio approaches $\pi/2$.The limiting speed for the curve, $\sqrt{R\mu g}$, is 71.3005 km/h; a straight stop from it takes 25.0000 m, while on the curve the limit is $\pi R/4=39.2699$ m, a ratio of $\pi/2=$ **1.570796** ——exactly at the limit, however, all the friction is used sideways and braking cannot begin; $\pi/2$ is where the ratio heads when the limit is approached from below.

5. The Separator

> **Observation 5.1**
>
> **the separator is whether braking and turning friction are treated as adding like numbers or as a quantity with direction.The friction limit applies to the combined length, so the fraction left for turning is $\sqrt{1 - b^2}$, not $1 - b$——a little braking is cheap and hard braking is dear. That stopping while turning takes $\sin^{-1} y/y$ times as far comes from the same circle.**

> **Observation 5.2**
>
> **beside Paper 453.It treated a banked curve on ice as a point mass and found the band of safe speeds with sideways friction alone——this paper adds longitudinal friction to the same point mass. While friction is used for turning, there is that much less for stopping.**

> **Observation 5.3**
>
> **one root, under Paper 300.The straight and the curved stopping distance come from the same friction circle——only the sideways use y separates them, and as $y \to 0$ the ratio returns to 1.**

> **Observation 5.4**
>
> **what is not said.No driving advice is given. Real tyres have different limits along and across, load transfers and anti-lock systems act, and none of these is included——what is shown is the form, a limit on a quantity with direction, and the closed form that follows.**

6. Roll Call and Closing

1. **half braking leaves 0.866025 for turning** (Computation 2.1): **addition says 0.5.**
2. **nine-tenths braking leaves 0.435890** (Computation 2.1): **56.4110 per cent of the turning grip gone.**
3. **30 per cent braking costs 4.6061 per cent** (Observation 2.2): **the first braking is cheap.**
4. **the turning radius grows 1.154701- and 2.294157-fold** (Computation 3.1): **at half and nine-tenths.**

5. **stopping while turning takes $(R/2)\sin^{-1}(v^2/(R\mu g))$** (Computation 4.1): **a closed form.**
6. **19.6714 m on a 50 m curve at 60 km/h** (Observation 4.2): **1.111163 times a straight stop; the core.**
7. **the ratio is $\sin^{-1} y/y$** (Computation 4.1): **1.047198, 1.244188, 1.443694.**
8. **near the limit, $\pi/2 = 1.570796$** (Observation 4.3): **exactly at the limit, braking cannot begin.**
9. **integration and closed form agree to 5.0×10^{-12}** (Computation 4.1): **two routes.**
10. **the separator is addition versus direction** (Observation 5.1): **the limit applies to the combined length.**

Closing: **Braking at half is easily taken to halve the margin for turning.Tyre friction shares one circle between braking and turning——using half the friction for braking still leaves 0.866025 for turning, and 30 per cent braking costs only 4.6061 per cent of the turning grip; but nine-tenths leaves only 0.435890. At 60 km/h with friction coefficient 0.8, the smallest radius one can turn grows 1.154701-fold with half braking and 2.294157-fold with nine-tenths.Stopping while turning takes $\sin^{-1} y/y$ times the distance of a straight stop——on a 50 m curve at 60 km/h the car stops in 19.6714 m, 1.111163 times the 17.7034 m of a straight stop. As the sideways use y approaches 1 the ratio approaches $\pi/2 = 1.570796$; exactly at the limit, all the friction is used sideways and braking cannot begin.The separator is whether braking and turning friction add like numbers or form a quantity with direction——the limit applies to the combined length; under Paper 300 the straight and curved stopping distances come from one circle and share a root.Placed among the earlier papers——Paper 453 found the band of safe speeds with sideways friction alone; this paper adds longitudinal friction to the same point mass.To be honest——the car is a point mass, the friction circle is idealised, and load transfer, anti-lock systems, tyre elasticity and banking are not included. The friction coefficient is representative, and this is not driving advice.**

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