The Earliest Sunset Comes Half a Month Before the Winter Solstice: the Gap Between Sundial and Clock Is the Sum of Two Causes

— the equation of time is the sum of an eccentricity part (amplitude **7.6550** minutes) and an obliquity part (amplitude **9.8664** minutes) — changing by **0.4975** minutes a day around the winter solstice, it brings the earliest sunset at latitude **35.7** degrees north **15.61** days before the solstice — [Paper 596]

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— *From thinking to syncing.* —

Abstract

The earliest sunset of the year is easily taken to fall on the winter solstice, the shortest day.What this paper shows is that the gap between sundial and clock (the equation of time) is the sum of two causes, the eccentricity of the orbit and the tilt of the ecliptic, and that around the solstice it changes by about 0.5 minutes a day, so the earliest sunset moves half a month before the solstice and the latest sunrise half a month after.No new theorem or law is claimed. First, the equation of time has two peaks and two troughs of different sizes——with eccentricity 0.0167 and obliquity 23.44 degrees, the maximum is 16.4187 minutes and the minimum -14.2455 minutes, the smaller peak and trough are $+3.6841$ and -6.5099 minutes, and it passes through zero four times a year (Section 2). **Second, the equation of time is the sum of two causes——the eccentricity part has amplitude 7.6550 minutes (7.6547 from the first-order $2e$), the obliquity part 9.8664 minutes (9.8634 from the first-order $\tan^2(\varepsilon/2)$), and their sum matches the whole to within 4.1×10^{-13} minutes** (Section 2). **Third, taking the causes one at a time changes the shape——with obliquity alone the peak and trough are equal at ± 9.8664 minutes and there are four zeros a year; with eccentricity alone they are ± 7.6550 minutes with two zeros. Large and small peaks alternate because a yearly and a half-yearly period overlap** (Section 3). **Fourth, turning the perihelion by 90 degrees changes the shape, but by 180 degrees does not——at 90 degrees the extremes become 14.5793 and -16.6434 minutes; at 180 degrees nothing changes, because the obliquity part has a half-year period** (Section 3). **Fifth, and this is the core.At latitude 35.7 degrees north, the earliest sunset comes 15.61 days before the winter solstice and the latest sunrise 16.07 days after ——the equation of time at the solstice is $+1.6951$ minutes and falls by 0.4975 minutes a day. Solar noon arrives later every day, so the shortest day and the earliest sunset fall on different days** (Section 4). **Sixth, the separator is whether one looks at the length of the day or also at the time of its middle (solar noon)——day length alone bottoms out at the solstice; when solar noon moves, the extremes of sunrise and sunset separate onto different days** (Section 5).

Keywords: equation of time, sunset, winter solstice, Kepler's equation, obliquity of the ecliptic.

Scope of this paper (scope note): **No new theorem or law is claimed——the equation of time, Kepler's equation and the equation of centre, conversion from ecliptic longitude to right ascension, and the sunrise equation are all standard.The orbital elements are representative——eccentricity 0.0167, obliquity 23.44 degrees and longitude of perihelion 282.94 degrees, with no secular change.Sunrise and sunset are computed from geometry alone——as in Paper 414, in local mean time at latitude 35.7 degrees north, without refraction or the sun's apparent radius.No calendar dates are given ——days are counted from perihelion and from the solstice.The history of calendars and clocks is not treated.Relation to earlier papers**: **Paper 414 showed that an hour of the old unequal-hour system varies by a factor of 1.5 with the season, and stated in its scope note: "the equation of time is not included; the difference between apparent and mean solar time is not the subject"——this paper takes that seat and puts the equation of time into the same sunrise equation at latitude 35.7 degrees.Paper 535 derived the moon's libration in longitude from the eccentricity, $2e$——the eccentricity part of the equation of time here comes from the same $2e$ (the equation of centre in Kepler's equation).Paper 300 showed that sharing a root is decidable——the moon's libration in longitude and the sun's equation of time are two appearances of $2e$ met in separate places, with one source, the equation of centre; in that sense they share a root.**What is added is **splitting the equation of time into its two causes and checking each amplitude against its first-order form**, **setting out the shapes with one cause at a time and with the perihelion turned**, **finding the days of earliest sunset and latest sunrise counted from the solstice**, and **placing the separator on day length versus the time of solar noon**.

1. Notation and the Question

Setting: **let L be the sun's mean longitude, λ its true longitude and α its right ascension.The difference between sundial time (apparent solar time) and clock time (mean solar time), the equation of time E, is**

$$E = (L - \lambda) + (\lambda - \alpha) \approx -2e \sin M + \tan^2 \frac{\varepsilon}{2} \sin 2\lambda,$$

the first term arising because the sun does not move uniformly along the ecliptic in a non-circular orbit (eccentricity e, mean anomaly M), the second because the ecliptic is tilted to the equator (obliquity ε).

Observation 1.1

the earliest sunset of the year is easily taken to fall on the winter solstice, the shortest day.If the day is shortest, sunset should be earliest——whether day length and sunset time bottom out on the same day is another question.

Question: **does the earliest sunset fall on the winter solstice?**

2. Two Causes

Computation 2.1: with eccentricity 0.0167, obliquity 23.44 degrees and longitude of perihelion 282.94 degrees, Kepler's equation was solved and the equation of time found through the year in steps of 0.01 days.

peak or trough (days from perihelion)	equation of time
39.01 days	-14.2455 minutes
130.84 days	$+3.6841$ minutes
203.91 days	-6.5099 minutes
303.96 days	$+16.4187$ minutes

The range is 30.6642 minutes, and the zeros fall at 102.28, 161.00, 241.12 and 355.95 days from perihelion, four times a year.The eccentricity part has amplitude 7.6550 minutes (7.6547 from the first-order $2e$) and the obliquity part 9.8664 minutes (9.8634 from the first-order $\tan^2(\varepsilon/2)$); the sum of the two differs from the whole by at most 4.1×10^{-13} minutes.

> **Observation 2.2**
>
> the equation of time is the sum of two causes.The eccentricity part comes from the sun moving faster near perihelion; the obliquity part from uniform motion along the ecliptic becoming uneven when projected on the equator——each agrees well with its first-order form, and together they make the whole.

3. One Cause at a Time

Computation 3.1: the same computation with one cause removed, and with the perihelion turned.

case	maximum	minimum	zeros
both	$+16.4187$ min	-14.2455 min	4
obliquity only (eccentricity 0)	$+9.8664$ min	-9.8664 min	4
eccentricity only (obliquity 0)	$+7.6550$ min	-7.6550 min	2
perihelion turned 90 degrees	$+14.5793$ min	-16.6434 min	4
perihelion turned 180 degrees	$+16.4187$ min	-14.2455 min	4

Observation 3.2

large and small peaks alternate because two periods overlap.The obliquity part has a half-year period with equal peaks and troughs; the eccentricity part has one of each per year——added together, one peak reinforces and the other cancels; the difference between $+16.4187$ and $+3.6841$ minutes comes from that.

Observation 3.3

turning the perihelion by half a turn leaves the shape unchanged.The obliquity part has a half-year period, so on days counted from perihelion a 180-degree turn lands on the same shape——a 90-degree turn changes the overlap, giving a maximum of 14.5793 and a minimum of -16.6434 minutes.

4. Around the Winter Solstice

Computation 4.1: **at latitude 35.7 degrees north, as in Paper 414, sunrise and sunset (local mean time, geometry only) were found with the equation of time included.The winter solstice (longitude 270 degrees) falls 352.54 days from perihelion (on the side of the previous year).**

	days from solstice	time
shortest day	$+0.00$ days	
earliest sunset	-15.61 days	16.6952 h (solstice 16.7616 h)
latest sunrise	$+16.07$ days	7.2505 h (solstice 7.1819 h)

The earliest sunset is 3.9798 minutes earlier than at the solstice, and the latest sunrise 4.1116 minutes later.The equation of time at the solstice is +1.6951 minutes and falls by 0.4975 minutes a day.

> **Observation 4.2**
>
> this is the core.Around the solstice the day length barely changes, but the equation of time falls by about 0.5 minutes a day, so solar noon by the clock moves about 0.5 minutes later each day——before the minimum of day length (the solstice), the later noon pushes sunset later, so the earliest sunset comes before the solstice; for sunrise it is the reverse, and the latest sunrise comes after.

5. The Separator

> **Observation 5.1**
>
> the separator is whether one looks at day length or also at the time of solar noon.Day length alone bottoms out at the solstice. When solar noon moves, the extremes of sunrise and sunset separate onto different days on either side of the solstice——“the solstice has the earliest sunset” is right only if solar noon does not move.

> **Observation 5.2**
>
> taking the seat left by Paper 414.It showed with the sunrise equation at latitude 35.7 degrees that an unequal hour varies by a factor of 1.5 with the season, and set aside the equation of time in its scope note ——this paper puts the equation of time into the same equation and shows that the extremes of sunrise and sunset move half a month from the solstice.

> **Observation 5.3**
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> one root with Paper 535, in the sense of Paper 300.Paper 535 derived the moon’s libration in longitude from $2e$; the eccentricity part of the equation of time comes from the same $2e$——two appearances of $2e$, met in separate places, the visible area of the moon and the gap between sundial and clock, have one source, the equation of centre in Kepler’s equation.

Observation 5.4

what is not said.No calendar dates are given and the history of clocks is not treated. Without refraction and the sun's apparent radius, real sunrise and sunset times differ from these——what is shown is the form, the extremes of sunrise and sunset moving half a month from the solstice, and the reason for it.

6. Roll Call and Closing

1. **the equation of time reaches 16.4187 and -14.2455 minutes** (Computation 2.1): **four zeros a year.**
2. **eccentricity part, amplitude 7.6550 minutes** (Computation 2.1): **first-order $2e$ gives 7.6547.**
3. **obliquity part, amplitude 9.8664 minutes** (Computation 2.1): **first-order gives 9.8634.**
4. **sum and whole agree to 4.1×10^{-13} minutes** (Computation 2.1): **the sum of two causes.**
5. **obliquity alone gives four zeros, eccentricity alone two** (Computation 3.1): **half-year and year periods.**
6. **perihelion turned 90 degrees: 14.5793 and -16.6434 minutes** (Observation 3.3): **unchanged at 180.**
7. **earliest sunset 15.61 days before the solstice** (Observation 4.2): **the core.**
8. **latest sunrise 16.07 days after the solstice** (Computation 4.1): **the extremes separate.**
9. **the equation of time falls 0.4975 minutes a day at the solstice** (Computation 4.1): **solar noon moves later daily.**
10. **the separator is day length versus solar noon** (Observation 5.1): **sharing $2e$ with Paper 535.**

Closing: **The earliest sunset of the year is easily taken to fall on the winter solstice, the shortest day.The gap between sundial and clock, the equation of time, is the sum of two causes——with eccentricity 0.0167 and obliquity 23.44 degrees it reaches 16.4187 and -14.2455 minutes, the eccentricity part (amplitude 7.6550 minutes) and the obliquity part (amplitude 9.8664 minutes) adding to the whole. A yearly and a half-yearly period overlap, so large and small peaks alternate; turning the perihelion by 90 degrees changes the extremes to 14.5793 and -16.6434 minutes, and by 180 degrees changes nothing.Around the solstice the equation of time falls by 0.4975 minutes a day——solar noon by the clock moves later every day, so at latitude 35.7 degrees north the earliest sunset comes 15.61 days before the solstice and the latest sunrise 16.07 days after. The shortest day and the earliest sunset are not the same day.The separator is whether one looks at day length or also at solar noon——day length alone bottoms out at the solstice; when noon moves, the extremes of sunrise and sunset separate on either side of it.Placed among the earlier papers——this takes the seat for the equation of time that Paper 414 left in its scope note. The eccentricity part comes from the same equation of centre as the $2e$ with which Paper 535 derived the moon's libration in longitude; as two appearances of $2e$ met in separate places, they share a root in the sense of Paper 300.To be honest——the orbital elements are representative with no secular change, and sunrise and sunset come from geometry**

alone, without refraction or the sun's apparent radius. No calendar dates are given, and the history of clocks is not treated.

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