Around a Well the Water Table Falls Only as the Logarithm of Distance, and Keeps Falling as the Logarithm of Time

— the drawdown decreases by only **3.6647** m for each tenfold increase in distance, and with pumping it keeps falling by **1.8323** m for each tenfold increase in time — in three-dimensional flow it settles to a steady **0.79577** m — [Paper 599]

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— From thinking to syncing. —

Abstract

Pumping a well lowers the water table around it, and it is easily taken that the effect fades a little way off.What this paper shows is that in a horizontally spreading aquifer (two-dimensional flow) the drawdown decreases only as the logarithm of distance and, with continued pumping, keeps falling as the logarithm of time without ever settling, and that one dimension more, in three-dimensional flow, changes this behaviour.No new theorem or law is claimed. First, a tenfold distance reduces the drawdown by only a fixed amount——3.6647 m by the steady Thiem formula: 13.5555 m at the well face, 9.8909 m at 1 m, 6.2262 m at 10 m and 2.5615 m at 100 m (Section 2). **Second, in three dimensions it falls to a tenth for each tenfold distance——for flow spreading spherically from a point, 7.9577 m at 1 m, 0.79577 m at 10 m and 0.079577 m at 100 m** (Section 2). **Third, distant wells act on each other——two wells 50 m apart pumping equally raise each other's drawdown by a factor of 1.270346** (Section 2). **Fourth, and this is the core.With continued pumping, the water table keeps falling as the logarithm of time and never settles——the drawdown 10 m away grows by 1.8323 m for each tenfold time, from 5.3284 m at 1 hour to 12.6572 m at 10000 hours** (Section 3). **Fifth, in three dimensions it settles——for point flow under the same conditions, the drawdown 10 m away is 0.77212 m at 1 hour and 0.79341 m at 100 hours, approaching the steady 0.79577 m without passing it** (Section 3). **Sixth, the separator is the dimension of the flow——in two dimensions both distance and time act logarithmically and there is no steady state; in three, drawdown goes inversely with distance and settles** (Section 4).

Keywords: well, groundwater, Thiem equation, Theis equation, aquifer.

Scope of this paper (scope note): No new theorem or law is claimed——the Thiem formula, the Theis formula, the Cooper--Jacob approximation and the formula for flow from a point are all standard.The aquifer is uniform and horizontal, confined above and below by impermeable layers.The numbers are representative——hydraulic conductivity 10^{-4} m/s, thickness 10 m, storativity 10^{-4}, pumping rate 0.01 m^3/s, well radius 0.1 m.A radius of influence of 500 m is set as the boundary of the steady formula——as Section 3 shows, in two dimensions there is in truth no steady state.Recharge of groundwater by rain, the edges of the aquifer and losses in the well itself are not included.No well is designed.Relation to earlier papers: Paper 574 showed that lowering the water table by 1 m raises the boundary between sea water and fresh water by 40 m——this paper looks at how the drawdown made by pumping spreads in distance and time; at a coastal well, the drawdown here lifts the bottom of the fresh water 40 times over, as in 574.Paper 315 showed that which dimension is special depends on the question——here, two dimensions are the boundary of logarithmic behaviour.What is added is giving the steady drawdown at four distances beside the three-dimensional case, finding the factor by which two wells act on each other, computing the Theis well function by two routes, series and integral, and showing that the water table keeps falling as the logarithm of time, and placing the separator on the dimension of the flow.

1. Notation and the Question

Setting: a well pumps Q per second from a horizontal aquifer of transmissivity T and storativity S.The drawdown at distance r is given, in the steady state, by the Thiem formula, and at time t after pumping starts by the Theis formula

$$s = \frac{Q}{2\pi T}\ln\frac{R}{r}, \qquad s = \frac{Q}{4\pi T}W(u), \quad u = \frac{r^2 S}{4Tt},$$

(R the radius of influence, W the well function), with $W(u) \approx -\gamma - \ln u$ for small u (Cooper--Jacob).For comparison, pumping from a point in a three-dimensional medium of conductivity K gives $s = Q/(4\pi K r)$.

Observation 1.1

pumping a well lowers the water table around it, and it is easily taken that the effect fades a little way off.Like sound or light, it should weaken quickly with distance——whether that holds for water spreading horizontally underground is another question.

Question: how does the drawdown of a well spread in distance and in time?

2. Distance

Computation 2.1: with hydraulic conductivity 10^{-4} m/s and thickness 10 m (transmissivity 10^{-3} m^2/s), pumping rate 0.01 m^3/s and radius of influence 500 m, $Q/(2\pi T) = 1.591549$ m.

distance	two dimensions (Thiem)	three dimensions (point)
0.1 m (well face)	**13.5555** m	
1 m	9.8909 m	**7.9577** m
10 m	6.2262 m	0.79577 m
100 m	2.5615 m	0.079577 m

> **Observation 2.2**
>
> in two dimensions the drawdown decreases by only **3.6647** m for each tenfold distance.In three dimensions it falls to a tenth——in a horizontally spreading aquifer water gathers from around a circle, so a tenfold radius widens the path only tenfold, and the drawdown lingers far off.

Computation 2.3: when two wells 50 m apart pump equally, each well's own drawdown of 13.5555 m at its face gains 3.6647 m from the other.The drawdown grows by a factor of **1.270346**.

3. Time

Computation 3.1: the well function $W(u)$ was computed by two routes, series and integral; at five values of u from 10^{-4} to 1 they differ by a relative 1.7×10^{-14} or less.With storativity 10^{-4}, $Q/(4\pi T) = 0.795775$ m, and each tenfold time adds **1.8323** m of drawdown.

10 m away	u	Theis	three dimensions (point)
1 hour	6.944×10^{-4}	5.3284 m	0.77212 m
10 hours	6.944×10^{-5}	7.1603 m	0.78829 m
100 hours	6.944×10^{-6}	8.9925 m	0.79341 m
1000 hours	6.944×10^{-7}	10.8249 m	
10000 hours	6.944×10^{-8}	12.6572 m	steady 0.79577 m

The Cooper--Jacob approximation agrees with the Theis formula to a relative 1.0×10^{-4} where u is below 10^{-3}.

> **Observation 3.2**
>
> this is the core.In two dimensions, with continued pumping the water table keeps falling as the logarithm of time and has not stopped at 10000 hours——each tenfold time lowers it by 1.8323 m. The region from which water is drawn keeps widening with time, and however far it widens, there is no distance at which the flow exactly balances.

> **Observation 3.3**
>
> in three dimensions it settles.For point flow the drawdown is 0.77212, 0.78829 and 0.79341 m, approaching the steady 0.79577 m without passing it——in three dimensions the path widens faster with distance, and the region from which water is drawn balances while still finite.

4. The Separator

> **Observation 4.1**
>
> the separator is the dimension of the flow.In two dimensions both distance and time act logarithmically and there is no steady state; in three, drawdown goes inversely with distance and settles——“the effect fades a little way off” and “it will settle even if pumping continues” are statements about three-dimensional flow.

Observation 4.2

beside Paper 574.It showed that on a coast, lowering the water table by 1 m raises the bottom of the fresh water by 40 m——since the drawdown here decreases only logarithmically with distance, even an inland well lowers the coastal water table, and that acts on the bottom of the fresh water 40 times over.

Observation 4.3

beside Paper 315.It showed that which dimension is special depends on the question——for the question here, whether drawdown settles, two dimensions are the boundary; the two-dimensional logarithm sits between the one-dimensional straight line and the three-dimensional inverse.

Observation 4.4

what is not said.No well is designed; recharge by rain, aquifer edges and losses in the well are not included, and real aquifers balance somewhere through recharge from rain and rivers——what is shown is the form: in a uniform, unbounded horizontal aquifer, both distance and time act logarithmically.

5. Roll Call and Closing

1. **3.6647 m less for each tenfold distance** (Observation 2.2): **the two-dimensional steady state.**
2. **13.5555 m at the well face** (Computation 2.1): **still 2.5615 m at 100 m.**
3. **in three dimensions, 7.9577 m at 1 m and 0.079577 m at 100 m** (Computation 2.1): **a tenth for each tenfold.**
4. **two wells 50 m apart, a factor of 1.270346** (Computation 2.3): **acting on each other from afar.**
5. **the two routes to the well function agree to 1.7×10^{-14}** (Computation 3.1): **series and integral.**
6. **1.8323 m more for each tenfold time** (Observation 3.2): **the core.**
7. **5.3284 m at 1 hour, 12.6572 m at 10000 hours** (Computation 3.1): **not stopping.**
8. **three dimensions settle at a steady 0.79577 m** (Observation 3.3): **without passing it.**
9. **Cooper--Jacob agrees with Theis to 1.0×10^{-4}** (Computation 3.1): **for $u < 10^{-3}$.**
10. **the separator is the dimension of the flow** (Observation 4.1): **two dimensions are the boundary.**

Closing: **Pumping a well lowers the water table around it, and it is easily taken that the effect fades a little way off and settles in time.In a horizontally spreading aquifer neither holds——pumping 0.01 m^3 per second from an**

aquifer 10 m thick, the drawdown decreases by only 3.6647 m for each tenfold distance, from 13.5555 m at the well face to 2.5615 m still at 100 m, and two wells 50 m apart raise each other's drawdown by a factor of 1.270346. Pumping on, the water table 10 m away falls by 1.8323 m for each tenfold time, from 5.3284 m at 1 hour to 12.6572 m at 10000 hours, and does not stop.In three-dimensional flow the behaviour changes——from a point, drawdown goes inversely with distance, 7.9577 m at 1 m becoming 0.079577 m at 100 m, and settles in time to a steady 0.79577 m.The separator is the dimension of the flow——in two dimensions both distance and time act logarithmically and there is no steady state.Placed among the earlier papers——on the coast of Paper 574, this drawdown lifts the bottom of the fresh water 40 times over; as an instance of Paper 315's special dimension depending on the question, two dimensions are the boundary for whether the water table settles.To be honest——the aquifer is uniform, horizontal and unbounded, without recharge from rain or rivers or losses in the well; real aquifers balance somewhere through recharge. The numbers are representative, and no well is designed.

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