The Sexagenary Cycle Has Sixty Pairs, Not a Hundred and Twenty

— Advancing the ten heavenly stems and the twelve earthly branches by one each year, 120 pairs are possible but only 60 occur, repeating every 60 years — pairing the Maya 260-day and 365-day counts, whose greatest common divisor is 5, only 18980 of 94900 pairs occur — [Paper 605]

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— *From thinking to syncing.* —

Abstract

The sexagenary cycle names years by pairing one of ten heavenly stems (jia, yi, bing and so on) with one of twelve earthly branches (zi, chou, yin and so on).Advancing both by one each year, how many pairs occur is counted.No new theorem or law is claimed. First, the pairs that occur are 60, not 120——collecting pairs over 120 years, 60 occurred (share 0.500000); the cycle repeats after 60 years and no new pair appears afterwards. Jia--zi comes again only after 60 years, the return known as kanreki in Japan (Section 2). **Second, only pairs of matching parity occur ——numbering the stems and branches from 0, all 60 pairs that occurred have indices of the same parity, and there are exactly 60 such pairs. The pair of jia (0) and chou (1) never comes** (Section 2). **Third, and this is the core.The separator is the greatest common divisor of the two periods——each year both indices advance by one, so the difference of the indices keeps its remainder on division by 2, the greatest common divisor of 10 and 12. With greatest common divisor 1 every pair occurs; with g, only $1/g$ of them** (Section 3). **Fourth, in the Maya calendar only a fifth occur——the pairs of 13 numbers and 20 day names (the tzolk'in) have greatest common divisor 1, and all 260 occur. The pairs of that 260-day count and the 365-day count have greatest common divisor 5; only 18980 of 94900 occur (share 0.200000), repeating after 18980 days (52 years of 365 days)** (Section 3). **Fifth, three pairwise coprime periods make everything occur——the three periods of 28, 19 and 15 years (the Julian period) have greatest common divisor 1 in every pair, and in 7980 years all 7980 triples occur once each** (Section 3). **Sixth, two routes agree——the number of pairs collected by running through the years and the number counted only by the condition "the difference of the indices leaves 0 on division by the greatest common divisor" agreed for the sexagenary cycle and for the Maya calendar** (Section 3). **Keywords**: sexagenary cycle, Chinese remainder theorem, greatest common divisor, calendar, Calendar Round.

Scope of this paper (scope note): **No new theorem or law is claimed——the theorem fixing a number from two remainders (the Chinese remainder theorem), that the sexagenary cycle repeats after 60 years, that the Maya calendar round is 52 years, and the Julian period (Scaliger, 1583) are all known.The history of calendars is not treated——reforms, leap days, regional differences, the origin of the stems and branches, and how the Maya count is aligned to present dates are not the subject.Conventions of year numbering are not discussed——all indices start from 0, and only how periods combine is counted.Relation to earlier papers**: **Paper 319 showed that a calendar is three stacked approximations fitting human numbers to natural periods——this paper is not about approximation; it counts how many pairs occur when two or three periods fixed by people are combined.**What is added is **setting the sexagenary cycle, the tzolk'in, the calendar round and the Julian period side by side under one count**, **matching the route of running through the years with the route of counting by the condition**, and **placing the separator on the greatest common divisor of the periods and checking in numbers that the share occurring is $1/g$**.

1. Notation and the Question

Setting: **number the stems jia, yi, bing, ding, wu, ji, geng, xin, ren, gui from 0 to 9, and the branches zi, chou, yin, mao, chen, si, wu, wei, shen, you, xu, hai from 0 to 11.The pair of year y (with $y = 0$ the year jia--zi) is (a, b), the remainders of y on division by 10 and by 12.**

In general: **when two counts with periods m_1, m_2 advance by one together, the pairs $(y \bmod m_1, y \bmod m_2)$ that occur are counted.**

Question: **of the $m_1 m_2$ possible pairs, how many occur, and what decides it?**

2. Counting by Running Through the Years

Computation 2.1: **pairs were collected by running through the years (up to three cycles, confirming that no new pair appears after one cycle).**

periods combined	possible pairs	pairs occurring	share	cycle
stems and branches (**10** and **12**)	**120**	**60**	**0.500000**	**60** years
tzolk'in (**13** and **20**)	**260**	**260**	**1.000000**	**260** days
calendar round (**260** and **365** days)	**94900**	**18980**	**0.200000**	**18980** days
Julian period (**28**, **19**, **15** years)	**7980**	**7980**	**1.000000**	**7980** years

Observation 2.2

the sexagenary cycle has only 60 pairs.Jia--zi comes again after 60 years, and in between each of the 60 pairs occurs once——half of the 120 candidates never come, however many years pass.

Observation 2.3

the 60 pairs that occur are those whose indices have the same parity.Jia--zi $(0,0)$, yi--chou $(1,1)$, bing--yin $(2,2)$ follow in turn, and after gui--you $(9,9)$ comes jia--xu $(0,10)$. Pairs of matching parity number $5\times 6+5\times 6=60$ and coincided exactly with those that occurred——jia--chou $(0,1)$ never comes, since its parities do not match.

3. The Separator

Observation 3.1

this is the core: the separator is the greatest common divisor of the two periods.a is the remainder of y on division by 10, so it has the same parity as y (10 being divisible by 2); likewise b has the same parity as y (12 being divisible by 2)——so a and b always have the same parity. 2 is the greatest common divisor of 10 and 12, and in general a, b and y all leave the same remainder on division by the greatest common divisor g.

Computation 3.2: **independently of running through the years, pairs were counted only by the condition "$a-b$ leaves 0 on division by the greatest common divisor g".**

periods	greatest common divisor g	pairs meeting the condition	least common multiple
10 and 12	2	60	60
13 and 20	1	260	260
260 and 365	5	18980	18980

The numbers counted by the condition agreed with those collected by running through the years, for the sexagenary cycle and for the calendar round.Each equals the least common multiple m_1m_2/g, a share $1/g$ of the candidates.

Observation 3.3

every pair meeting the condition occurs.For a pair (a, b) of matching parity, there is exactly one year among 60 that leaves a on division by 10 and b on division by 12——this is the theorem fixing a number from two remainders (the Chinese remainder theorem). When the greatest common divisor is 1, the condition disappears and every pair occurs.

Observation 3.4

the Maya calendar is the same mechanism in two stages.13 and 20 have greatest common divisor 1, so all 260 pairs line up in the 260-day tzolk'in. That tzolk'in and the 365-day count have greatest common divisor 5, so only a fifth, 18980 pairs, occur, repeating after 18980 days, that is 52 years of 365 days——in the first stage nothing vanishes; in the second, four fifths vanish. The only difference is the greatest common divisor.

Observation 3.5

the Julian period is built so that nothing vanishes.28, 19 and 15 have greatest common divisor 1 in every pair, and in 7980 years all 7980 triples occur once each——Scaliger built it as the length within which the triple of indices fixes a single year.

Observation 3.6

beside Paper 319.Paper 319 found that the Julian calendar drifts by one day in 128 years and the Gregorian by one day in 3226 years, and showed a calendar to be three stacked approximations——there the issue was approximating natural periods with human numbers. Here all periods are integers fixed by people, with no approximation, and whether pairs vanish is decided only by the greatest common divisor of those integers.

Observation 3.7

what is not said.The history of calendars, reforms, leap days, the origin of the stems and branches, and how the Maya count is aligned to present dates are not treated——what is shown is that the number of pairs occurring when periods are combined is decided by the greatest common divisor, checked by two routes.

4. Roll Call and Closing

1. **60 of the 120 possible pairs occur** (Computation 2.1): **share 0.500000.**
2. **the cycle repeats after 60 years with no new pair** (Computation 2.1): **kanreki.**
3. **only pairs of matching parity occur** (Observation 2.3): **jia--chou never comes.**
4. **the separator is the greatest common divisor of the periods** (Observation 3.1): **the core.**
5. **pairs counted by the condition match those from the years** (Computation 3.2): **two routes.**
6. **the share occurring is $1/g$** (Computation 3.2): **as many as the least common multiple.**
7. **all 260 pairs of the tzolk'in occur** (Computation 2.1): **greatest common divisor 1.**
8. **the calendar round has 18980 of 94900** (Computation 2.1): **greatest common divisor 5, 52 years.**
9. **all 7980 triples of the Julian period occur** (Computation 2.1): **built so nothing vanishes.**
10. **decided by the greatest common divisor of integers, not by approximation** (Observation 3.6): **the difference from Paper 319.**

Closing: **The sexagenary cycle names years by advancing the ten stems and the twelve branches by one each year.There are 120 possible pairs, but only 60 occur——running through the years, the cycle repeats after 60 years and no new pair appears. Only pairs whose indices have the same parity occur, and jia--chou never comes.The separator is the greatest common divisor of the two periods——since both indices advance together, the difference of the indices keeps its remainder on division by the greatest common divisor. With greatest common divisor g, the pairs occurring are $1/g$ of the candidates, and the count from the years agreed with the count by the condition.The same mechanism is in the Maya calendar——13 and 20 have greatest common divisor 1 and all 260 pairs occur, while 260 and 365 days have 5 and only 18980 of 94900 occur. The Julian period, built from 28, 19 and 15 years, has greatest common divisor 1 in every pair, and all 7980 triples occur.Placed among the earlier papers——unlike the approximations of the calendar treated in Paper 319, all periods here are integers, and whether pairs vanish is decided by the greatest common divisor alone.To be honest——this is a count under a known theorem, and neither the history of calendars nor the alignment of the Maya count is touched.**

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