Only One Light Pattern in Four Can Be Switched Off on a 5×5 Lights Out Board

— On a 5×5 board where pressing a light flips it and its neighbours above, below, left and right, 2^{23} of the 2^{25} patterns can be switched off entirely — 3×3 can clear every pattern, 4×4 one in 16, 19×19 one in 65536, not monotone in the size — [Paper 606]

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— *From thinking to syncing.* —

Abstract

Lights Out is a game on a grid of lights in which pressing a light flips it and the lights above, below, left and right.It is counted which light patterns can be switched off entirely by pressing.No new theorem or law is claimed. First, on 5×5 one pattern in four can be switched off——elimination modulo two gives the rank 23 for the table of presses, so $2^{23} = 8388608$ patterns can be cleared, a share 0.250000 of all 2^{25}. Trying all 2^{25} ways of pressing and counting the patterns made gave the same 8388608 (Sections 2 and 3). **Second, there are ways of pressing that change nothing ——on 5×5 there are 3 ways, pressing 16 or 12 squares, that leave the board as it was. Because of them, the 2^{25} ways of pressing make each pattern 4 times, and the patterns made are cut to a quarter** (Section 4). **Third, and this is the core.The patterns that can be switched off are exactly those meeting each do-nothing press in an even number of lights——over all 2^{25} patterns, those cleared by trying everything and those meeting this condition agreed without exception. The pattern with only the top-left light on meets one do-nothing press in 1 light and cannot be cleared. The all-on pattern can, in 4 ways, each pressing 15 squares** (Section 4). **Fourth, the separator is the size of the board, and it is not monotone——for boards from 1 to 20, the dimension of the do-nothing presses is 4 for 4×4, 2 for 5×5, 8 for 9×9, 6 for 11×11, 4 for 14×14, 8 for 16×16, 2 for 17×17 and 16 for 19×19, and 0 (every pattern can be cleared) for the other 12 sizes** (Section 5). **Fifth, the number of quantities watched decides the share out of reach——in the fifteen puzzle one quantity is watched and half is reachable. On the 5×5 Lights Out board two are watched, and one pattern in four can be cleared** (Section 5). **Keywords**: Lights Out, linear algebra over GF(2), puzzle, null space, board size.

Scope of this paper (scope note): **No new theorem or law is claimed——solving Lights Out as linear equations modulo two, and that one pattern in four can be cleared on 5×5, are known (Anderson and Feil, 1998; Sutner, 1989).How the dimension depends on the board size is not treated——boards from 1 to 20 are eliminated and counted, without entering the theory of polynomials behind it.How to find the fewest presses is not treated——only for the all-on pattern are the 4 ways of pressing listed.Relation to earlier papers**: **Paper 601 showed with one watched quantity that half the positions of the fifteen puzzle can never be reached——here two quantities are watched, and one pattern in four can be cleared.Paper 602 showed that who wins at Nim is decided by addition without carries——pressing lights is that addition too: pressing a square twice cancels, and the order of pressing does not matter.**What is added is **matching elimination with an exhaustive trial of all 2^{25} ways of pressing on 5×5, checking the criterion for clearable patterns on all 2^{25} patterns**, and **listing the dimension of the do-nothing presses for boards from 1 to 20, showing that the separator is not monotone in the size**.

1. Notation and the Question

Setting: **each square of an $n \times n$ board has a light.Pressing a square flips its light and those above, below, left and right (where they are on the board).Writing a light pattern as a string of 0s and 1s, pressing adds, without carries, the string of lights that square flips.**

Two facts: **pressing the same square twice restores it, and the order of pressing does not change the result.So a way of pressing is fixed by which squares are pressed, and there are 2^{n^2} of them, pressing or not each of the n^2 squares.**

Question: **of the 2^{n^2} light patterns, how many can be switched off entirely, and how can one tell which?**

2. Counting by Elimination

Computation 2.1: **the n^2 strings flipped by pressing each square were eliminated modulo two to find the rank r .Exactly 2^r patterns can be cleared, and the do-nothing presses form a space of dimension $n^2 - r$.**

board	squares	rank	dimension of do-nothing presses	share clearable
3×3	9	9	0	1
4×4	16	12	4	1/16
5×5	25	23	2	1/4
9×9	81	73	8	1/256
11×11	121	115	6	1/64
14×14	196	192	4	1/16
16×16	256	248	8	1/256
17×17	289	287	2	1/4
19×19	361	345	16	1/65536

For the sizes not in the table, 1, 2, 3, 6, 7, 8, 10, 12, 13, 15, 18 and 20, the rank equals the number of squares and every pattern can be cleared.

3. Counting by Trying Everything

Computation 3.1: **independently of elimination, every way of pressing was tried and the patterns made were counted.**

board	ways of pressing	patterns made	share of all patterns
2×2	2^4	16	1.000000
3×3	2^9	512	1.000000
4×4	2^{16}	4096	**0.062500**
5×5	2^{25}	8388608	**0.250000**

Observation 3.2

the two routes agree.On all four boards, the number of patterns made equals 2^r——on 5×5 there are 2^{25} ways of pressing but only 8388608 patterns; the ways of pressing make each pattern 4 times.

4. Presses That Change Nothing

Computation 4.1: **on 5×5 the presses that leave the board as it was were taken from the elimination, and pressing them was checked to change nothing.There are 3 such presses (● marks a pressed square; rows from the top are separated by "/").**

16 squares: ·●●●·/●·●·●/●●·●●/●·●·●/·●●●·

12 squares: ●·●·●/●·●·●/·····/●·●·●/●·●·●

12 squares: ●●·●●/·····/●●·●●/·····/●●·●●

Computation 4.2: **over all 2^{25} patterns, those "meeting each do-nothing press in an even number of lights" numbered 8388608 and coincided exactly with the set cleared by trying everything.The all-on pattern can be cleared, in 4 ways, each pressing 15 squares. The pattern with only the top-left light on cannot.**

Observation 4.3

this is the core: the clearable patterns are those meeting each do-nothing press in an even number of lights.Pressing is symmetric: if pressing a flips b, pressing b flips a——so a pattern meeting a do-nothing press in an odd number of lights cannot be made by any pressing. The top-left-only pattern meets the second 12-square press in the top-left square alone; the overlap is 1, so it cannot be cleared.

Observation 4.4

the all-on pattern is cleared by four ways of pressing, each of 15 squares.Adding a do-nothing press to one way of pressing clears the same pattern, and the 4 ways turn into one another like this——pressing 15 of the 25 squares of 5×5 switches a fully lit board completely off.

5. The Separator

Observation 5.1

the separator is the size of the board, and it is not monotone.3×3 can clear every pattern, 4×4 one in 16, 5×5 one in four, 6 to 8 every pattern again, and 9×9 one in 256——larger boards are not harder to clear. Up to 20 the hardest is 19×19 with one in 65536, and 20×20 can clear every pattern.

Observation 5.2

beside Paper 601.Paper 601 showed with one watched quantity (the sum of the parities of the tiles' arrangement and of the blank's distance) that half the positions of the fifteen puzzle can never be reached——in Lights Out, each do-nothing press is a watched quantity. On 5×5 there are two independent ones and $1/2^{2}$ is reachable; on 19×19 there are sixteen and $1/2^{16}$.

Observation 5.3

beside Paper 602.Paper 602 showed that who wins at Nim is decided by addition without carries——pressing lights is the same addition. Pressing a square twice cancels, and the order does not matter, because there are no carries.

Observation 5.4

what is not said.The theory of how the dimension depends on the board size is not entered, and how to find the fewest presses is shown only for the all-on pattern——what is shown is that the two routes agree pattern by pattern on 5×5, how to tell the clearable patterns, and that the dimension is not monotone for boards up to 20.

6. Roll Call and Closing

1. **the rank on 5×5 is 23** (Computation 2.1): **2^{23} clearable patterns.**
2. **the clearable share is 0.250000** (Computation 3.1): **one in four.**
3. **trying all 2^{25} presses also gives 8388608** (Computation 3.1): **two routes.**

4. **3 presses change nothing** (Computation 4.1): **16, 12 and 12 squares.**
5. **clearable patterns meet them in even numbers of lights** (Observation 4.3): **the core.**
6. **the top-left light alone cannot be cleared** (Observation 4.3): **an overlap of 1.**
7. **the all-on pattern clears with 15 presses** (Observation 4.4): **in 4 ways.**
8. **4×4 one in 16, 19×19 one in 65536** (Computation 2.1): **not monotone.**
9. **12 sizes up to 20 clear every pattern** (Computation 2.1): **dimension 0.**
10. **the number of watched quantities decides the share out of reach** (Observation 5.2): **one in 601, two here.**

Closing: **In Lights Out, pressing a light flips it and its neighbours above, below, left and right.On a 5×5 board only one light pattern in four can be switched off entirely——elimination modulo two gives rank 23, so $2^{23} = 8388608$ patterns are clearable, agreeing with the patterns counted by trying all 2^{25} ways of pressing.The clearable patterns are those meeting each press that changes nothing in an even number of lights——there are 3 such presses, and this criterion agreed with trying everything on all 2^{25} patterns. The top-left light alone cannot be cleared, and the all-on board clears with 15 presses.The separator is the size of the board, and it is not monotone——3×3 clears every pattern, 4×4 one in 16, 19×19 one in 65536, and 20×20 every pattern.Placed among the earlier papers——the fifteen puzzle of Paper 601 watches one quantity and reaches half; here two are watched and one in four is reached. Pressing is the addition without carries of Paper 602.To be honest——the theory of how the dimension depends on the size is not entered; only boards up to 20 were counted.**

References

1. Anderson, M., & Feil, T. (1998). Turning lights out with linear algebra. *Mathematics Magazine*.
2. Sutner, K. (1989). Linear cellular automata and the Garden-of-Eden. *The Mathematical Intelligencer*.
3. Yamagishi, Y. (2026). *Half of the Fifteen Puzzle's Positions Can Never Be Reached* [Paper 601]. Zenodo. 10.5281/zenodo.22805752
4. Yamagishi, Y. (2026). *Who Wins at Nim Is Decided by Addition Without Carries* [Paper 602]. Zenodo. 10.5281/zenodo.22805951

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