The Chance of Winning by Switching Doors Is Decided by How the Host Opens Them

— if the host knows where the car is and opens a losing door, switching wins $2/3$ — if the host opened at random and a goat happened to appear, $1/2$ — if, with two losing doors, the host opens a given one with probability q, it is $1/(1+q)$, and with $q = 0$ switching always wins — [Paper 611]

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— *From thinking to syncing.* —

Abstract

A car is behind one of three doors and goats behind the other two. The contestant picks one; the host opens one of the other two to show a goat and asks whether the contestant wants to switch (the Monty Hall problem).The chance of winning by switching is counted for each way the host may open a door.No new theorem or law is claimed. First, if the host knows, switching wins $2/3$——enumerating as fractions the cases of the car's position and the host's choice, switching wins $2/3$ and staying $1/3$. In 200000 random trials, switching won 0.666915 (Section 2). **Second, if the host opened without knowing, switching wins $1/2$——counting only the cases in which the host opens one of the other two doors at random and a goat happens to appear, switching and staying each win $1/2$. In random trials, of the 133400 that remained after removing those where the car was revealed, switching won 0.500435** (Section 2). **Third, and this is the core.The separator is not the door but the host's rule for opening——the same door opening in the same way to show the same goat gives different answers depending on whether the host knew. What the open door conveys is not where the goat is but the fact that the host chose it** (Section 3). **Fourth, a host with a habit moves the answer from $1/2$ to 1——suppose that with two losing doors the host opens door 2 with probability q. The chance of winning by switching when door 2 opens is $1/(1+q)$: 1 for $q = 0$, $4/5$ for $q = 1/4$, $4/7$ for $q = 3/4$ and $1/2$ for $q = 1$** (Section 3). **Fifth, with more doors the difference grows——with n doors, if the host opens $n-2$ losing doors, switching wins $(n-1)/n$, which is 0.99 for 100 doors** (Section 3). **Keywords**: Monty Hall problem, conditional probability, host's rule, Bayes' theorem, probability paradox.

Scope of this paper (scope note): **No new theorem or law is claimed——that switching wins $2/3$ (Selvin, 1975), and that the answer changes with the host's knowledge and rule (Gillman, 1992, and others; summarised in Rosenhouse's book of 2009) are known.Why people do not switch is not treated——only probabilities are counted, without entering the psychology of the decision.The host's rules are assumptions——"knows and opens a losing door", "opens at random without knowing" and "opens with habit q" are only set side by side; which one the actual show used is not stated.Relation to earlier papers**: **Paper 164 showed that "probability" has four addresses——here, with the same door opened, the address given by the host's rule changes the probability.Paper 607 showed that how often the first player wins at tic-tac-toe splits three ways by what is taken as equal——here too the answer changes with what is assumed about how the host opens.**What is added is **setting the three rules side by side under one enumeration, exactly as fractions**, **matching the two routes of fractions and random trials**, and **placing the separator on the host's rule for opening and showing that the habit q moves the answer from $1/2$ to 1**.

1. Notation and the Question

Setting: **the doors are $0, 1, 2$, and the car is placed behind any of them with equal probability. The contestant picks door 0.The host opens door 1 or 2. Here the case in which door 2 opens to show a goat is considered, and switching means moving to door 1.**

Three rules for the host: **(i) the host knows where the car is and opens a losing door, choosing equally if both are losing. (ii) the host does not know and opens door 1 or 2 equally (cases revealing the car are not counted). (iii) the host knows and opens a losing door, but with two losing doors opens door 2 with probability q.**

Question: **when door 2 opens to show a goat, what is the chance of winning by switching, and what decides it?**

2. Enumeration and Random Trials

Computation 2.1: **the cases of the car's position (three) and of the host's choice were enumerated as fractions, conditioning on door 2 opening to show a goat.**

host's rule	win by switching	win by staying
(i) knows and opens a losing door	$2/3$	$1/3$
(ii) opens at random without knowing	$1/2$	$1/2$

Computation 2.2: **independently of the enumeration, 200000 random trials were run for each rule.Under rule (i) switching won 0.666915; under rule (ii), of the 133400 trials remaining after removing those that revealed the car,**

switching won 0.500435.

> **Observation 2.3**
>
> the two rules give different answers.Under rule (i), if the contestant first picked a losing door (probability $2/3$), the host must open the other losing door, and switching always finds the car——under rule (ii), that the host showed a goat is itself chance, and it does not reduce the possibility that the first pick was right.

3. The Separator

> **Observation 3.1**
>
> this is the core: the separator is not the door but the host's rule for opening.Under rules (i) and (ii), the opened door and the goat seen are the same——what differs is only why the host chose that door. What the open door conveys is not the goat's position but the host's choice, and the probability changes with what the host knew in making it.

Computation 3.2: under rule (iii), the chance of winning by switching was counted as a fraction for each habit q.

habit q	win by switching	formula $1/(1+q)$
0	1	1
1/4	4/5	4/5
1/2	2/3	2/3
3/4	4/7	4/7
1	1/2	1/2

> **Observation 3.3**
>
> knowing the habit, the answer moves from $1/2$ to 1.A host with $q = 0$ always opens door 1 when both are losing. If that host opens door 2, it is because door 1 hides the car, and switching always wins——with $q = 1$, door 2 opening tells nothing and the answer is $1/2$. The familiar $2/3$ is the equal choice $q = 1/2$.

Observation 3.4

with more doors the difference grows.With n doors, if the host knows and opens $n-2$ losing doors, switching wins $(n-1)/n$: 0.9 for 10 doors, 0.99 for 100——the $1/n$ that the first pick was right does not move however many doors the host opens.

Observation 3.5

beside Paper 164 and Paper 607.Paper 164 showed that “probability” has four addresses, and Paper 607 that how often the first player wins at tic-tac-toe splits three ways by what is taken as equal——here the same door and goat give $1/2$, $2/3$ or 1 just by restating the assumption about the host’s rule. Before answering with a probability, one must ask for that rule.

Observation 3.6

what is not said.Why people do not switch is not treated, nor which rule the host of the actual show followed ——what is shown is the exact chance of winning by switching under each of three rules, confirmed by random trials.

4. Roll Call and Closing

1. **if the host knows, switching wins $2/3$** (Computation 2.1): **staying $1/3$.**
2. **random trials give 0.666915** (Computation 2.2): **two routes.**
3. **if the host opened without knowing, $1/2$** (Computation 2.1): **random trials give 0.500435.**
4. **the separator is the host’s rule for opening** (Observation 3.1): **the core.**
5. **the open door conveys the host’s choice** (Observation 3.1): **not the goat’s position.**
6. **with habit q, $1/(1+q)$** (Computation 3.2): **agreeing for five values of q.**
7. **with $q=0$ switching always wins** (Observation 3.3): **with $q=1, 1/2$.**
8. **with n doors, $(n-1)/n$** (Observation 3.4): **0.99 for 100 doors.**
9. **the first pick’s $1/n$ does not move** (Observation 3.4): **however many doors open.**
10. **ask for the rule before the probability** (Observation 3.5): **Papers 164 and 607.**

Closing: **A car is behind one of three doors; after the contestant picks one, the host opens another to show a goat.The chance of winning by switching is decided by how the host opens the doors——if the host knows where the car is and opens a losing door it is $2/3$; if the host opened without knowing and a goat happened to appear it is $1/2$; the enumeration by fractions and 200000 random trials agreed.The separator is not the door but the host's rule for opening——even with the same door and goat, the answer changes with what the host knew in choosing. With a habit of opening one of two losing doors with probability q, it is $1/(1+q)$, moving from $1/2$ to 1.With more doors the difference grows——opening $n-2$ losing doors out of n, switching wins $(n-1)/n$.Placed among the earlier papers——after the addresses of probability in Paper 164 and what is taken as equal in Paper 607, here the host's rule decides the probability.To be honest——the host's rules are assumptions, and the psychology of people's decisions is not treated.**

References

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