The Expected Number of Crossings of Buffon's Needle Does Not Change When the Needle Is Bent

— Dropping a needle of length $l \leq d$ on parallel lines spaced d apart, the chance of crossing is $2l/(\pi d)$, which is 0.636620 for $l = d$ — what does not depend on shape is the expectation, not the chance — [Paper 613]

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— *From thinking to syncing.* —

Abstract

Parallel lines spaced d apart are drawn on a floor, and a needle of length l is dropped at random. Asking the chance that it crosses a line is Buffon's needle problem.The chance of crossing and the expected number of crossings are counted as the length and shape of the needle change.No new theorem or law is claimed. First, for a short needle the chance of crossing is $2l/(\pi d)$——for $l = d$ the integral gives 0.636620; dropping 2 million needles at random gives 0.636781, from which π comes out as 3.140797 (Section 2). **Second, for a long needle the chance and the expectation part——for $l = 2d$ the chance of crossing is 0.837248 and the expected number of crossings $2l/(\pi d) = 1.273240$; for $l = 5d$ the chance is 0.936123 against an expectation of 3.183099. One needle crosses two or more lines, so the expectation keeps growing in proportion to the length while the chance never exceeds 1** (Section 2). **Third, and this is the core.The expected number of crossings does not change when the needle is bent——dropping 100000 broken lines of length L in random shapes, the expected number of crossings was 0.635280 for $L = d$ and 1.904310 for $L = 3d$, both matching the $2L/(\pi d)$ of a straight needle. A circle of diameter d (length πd) crosses exactly twice wherever it falls** (Section 3). **Fourth, the separator is whether one asks for the chance or for the expectation——the chance that a broken line of length d crosses is 0.395150, much smaller than the 0.636620 of a straight needle. The chance changes with the shape; the expectation is fixed by the length alone. The expectation is the sum of the contributions of the needle's small pieces, so it does not depend on how they are joined** (Section 3). **Keywords**: Buffon's needle, integral geometry, estimation of pi, Barbier's theorem, geometric probability.

> **Scope of this paper (scope note)**: **No new theorem or law is claimed——the chance that a needle crosses (Buffon, 1777), that the expected number of crossings of a curve depends only on its length (Barbier, 1860; Ramaley, 1969), and the integral geometry behind it (Santaló, 1976) are all known.The general theory of integral geometry is only mentioned ——the count is limited to parallel lines, needles and broken lines.It is not recommended as a way to compute π——even 2 million needles give only three correct digits.Relation to earlier papers**: **Paper 31 separated the dimensionless π into the π of a solid angle and the π of a period——the π here comes from $2/\pi$, the average of $|\sin\theta|$ over the needle's directions.**What is added is **matching the integral for the chance of crossing with random drops for four lengths**, setting **out in numbers how chance and expectation part for long needles**, and **placing the separator on chance versus expectation and confirming with broken lines that the expectation does not depend on shape**.

1. Notation and the Question

Setting: **infinitely many parallel lines are drawn on a floor, spaced d apart. A needle of length l is dropped with its centre and direction uniformly random.With θ the angle between needle and lines, the needle occupies a length $l|\sin\theta|$ in the direction across the lines.**

Question: **what are the chance that the needle crosses a line and the expected number of crossings, and what changes and what does not when the needle is long or bent?**

2. The Straight Needle

Computation 2.1: **with $d=1$, the chance of crossing was found by integrating over the angle, and 2 million needles were dropped at random for each length to count the chance of crossing and the expected number of crossings.**

length l	chance (integral)	chance (random)	expected crossings (random)	$2l/(\pi d)$
0.5	0.318310	0.318134	0.318134	0.318310
1	**0.636620**	0.636781	0.636781	0.636620
2	0.837248	0.836786	1.272425	1.273240
5	0.936123	0.936259	3.183575	3.183099

Working back from the random chance 0.636781 for $l=1$, $\pi=2l/(dP)$ came out as 3.140797.

Observation 2.2

for a short needle the chance and the expectation are the same.A needle with $l \le d$ cannot cross two lines, so it crosses 0 or 1 times and the expectation equals the chance——the $2/\pi$ in $2l/(\pi d)$ is the average of $|\sin\theta|$ over the needle's directions.

Observation 2.3

for a long needle the chance and the expectation part.For $l = 2d$, a chance of 0.837248 against an expectation of 1.273240; for $l = 5d$, 0.936123 against 3.183099——the expectation keeps growing in proportion to the length, while the chance only approaches 1. Since one needle crosses many lines, "does it cross" and "how many times does it cross" become different questions.

3. The Bent Needle

Computation 3.1: a length L was divided into 30 segments, bent at random between neighbouring segments, and 100000 such broken lines were dropped with random position and direction.

length L	expected crossings (random)	$2L/(\pi d)$	chance of crossing (random)
1	0.635280	0.636620	0.395150
3	1.904310	1.909859	0.917120

A circle of diameter d (length πd) crosses exactly twice wherever it falls, matching $2L/(\pi d) = 2$.

Observation 3.2

this is the core: the expected number of crossings does not change when the needle is bent.Dividing a bent needle into small pieces, the expected number of crossings of each piece is proportional to its length, and the expectation for the whole is their sum——the sum does not depend on how the pieces are joined, so straight, broken or circular, a length L gives $2L/(\pi d)$.

Observation 3.3

the separator is whether one asks for the chance or the expectation.The chance that a broken line of length d crosses is 0.395150, much smaller than 0.636620 for a straight needle. A bent needle more easily fits between lines, and when it crosses it may cross twice——the chance changes with shape; the expectation is fixed by length alone.

Observation 3.4

beside Paper 31.Paper 31 separated the dimensionless π into one from solid angles and one from periods ——the π of Buffon's needle comes from averaging the needle's direction over a half turn: the average of $|\sin\theta|$ is $2/\pi$, and the π of the circle enters there.

Observation 3.5

what is not said.The general theory of integral geometry is not entered, and it is not recommended as a way to compute π: 2 million needles give only three digits——what is shown is that chance and expectation part for long needles, and that the expectation does not depend on shape.

4. Roll Call and Closing

1. **chance of crossing for $l = d$: 0.636620** (Computation 2.1): **$2/\pi$.**
2. **random drops give 0.636781, and π as 3.140797** (Computation 2.1): **two routes.**
3. **for a short needle, chance equals expectation** (Observation 2.2): **it cannot cross two lines.**
4. **$l = 2d$: chance 0.837248, expectation 1.273240** (Computation 2.1): **they part.**
5. **$l = 5d$: chance 0.936123, expectation 3.183099** (Computation 2.1): **expectation proportional to length.**
6. **broken line $L = d$: expectation 0.635280** (Computation 3.1): **the same as straight.**
7. **broken line $L = 3d$: expectation 1.904310** (Computation 3.1): **again $2L/(\pi d)$.**
8. **a circle always crosses twice** (Computation 3.1): **length πd.**
9. **the expectation does not depend on shape** (Observation 3.2): **the core.**
10. **a broken line crosses with chance 0.395150** (Observation 3.3): **the separator is chance versus expectation.**

Closing: **A needle of length l is dropped on parallel lines spaced d apart.For a short needle the chance of crossing is $2l/(\pi d)$, the same as the expected number of crossings——0.636620 for $l = d$, and 0.636781 from 2 million needles. For long needles chance and expectation part: for $l = 5d$, a chance of 0.936123 against an expectation of 3.183099 .The expected number of crossings does not change when the needle is bent——dropping randomly broken lines of length L, the expectation matched $2L/(\pi d)$, and a circle crossed twice wherever it fell. The expectation is the sum of the contributions of small pieces and does not depend on how they are joined.The separator is whether one asks for the chance or the expectation——at the same length, a broken line crosses with chance 0.395150, unlike the 0.636620 of a straight needle.Placed among the earlier papers——beside the sources of π separated in Paper 31, it sets the $2/\pi$ that comes from averaging over directions.To be honest——the general theory of integral geometry is not entered; only parallel lines, needles and broken lines were counted.**

References

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