A Tsunami Slows as the Water Shallows, and Its Height Grows as the Inverse Fourth Root of the Depth

— A tsunami travelling at **198.057062** m/s (**713** km/h) over **4000** m of water slows to **9.902853** m/s in **10** m — solving the linear shallow-water equations numerically agrees on the slope to within **1.6** per cent — [Paper 616]

Yuuki YAMAGISHI (山岸 勇輝)
Yamagishi Complexity Torus Lab, Japan
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— *From thinking to syncing.* —

Abstract

How the speed of an ocean wave is set changes according to whether its wavelength is much longer or much shorter than the depth.The speed of a tsunami and the growth of its height in shallow water are obtained by two routes, formula and numerical computation.No new theorem or law is claimed. First, a tsunami's speed is fixed by the depth alone——a wave much longer than the depth travels at $\sqrt{gh}$: 198.057062 m/s (713 km/h) at 4000 m, 31.315571 m/s at 100 m and 9.902853 m/s at 10 m. Crossing 10000 km of 4000-m ocean takes 14.025139 hours (Section 2). **Second, solving the dispersion relation gives the same——solving the general relation for wave speed numerically, a tsunami with a 10-minute period at 4000 m has a wavelength of 117.947696 km and a speed 0.992540 times $\sqrt{gh}$. A wind wave with a 10-second period has a wavelength of 156 m, and its speed of 15.607768 m/s is fixed by the period alone, not the depth** (Section 2). **Third, and this is the core.As the water shallows, the height grows as the inverse $1/4$ power of the depth——the flow of energy carried by the wave (the square of the height times the speed) is kept, so the height grows as much as the speed falls (Green's law): 1.414214 times from 4000 m to 1000 m, 2.514867 times to 100 m and 4.472136 times to 10 m, while the wavelength shrinks in proportion to $\sqrt{h}$** (Section 3). **Fourth, numerical computation agrees with Green's law on the slope to within 1.6 per cent——for a sea shoaling from 4000 m to 40 m over 600 km, solving the linear shallow-water equations on a 6000-point grid gave height ratios of 1.422233 at 1000 m and 2.555027 at 100 m, relative differences from Green's law of 0.005671 to 0.015969. On the shelf beyond the slope (40 m) it gave 3.101584, 0.019193 below (the cause has not been checked)** (Section 4). **Fifth, the separator is whether the wavelength is longer than the depth——if longer, the speed is fixed by the depth alone, and the wave slows and grows taller as the water shallows. If shorter, the speed is fixed by the period alone and the wave does not feel the depth of the deep ocean** (Section 5). **Keywords**: tsunami, shallow-water wave, Green's law, wave speed, water depth.

Scope of this paper (scope note): **No new theorem or law is claimed——the shallow-water wave speed $\sqrt{gh}$, the law for the height when the depth changes (Green, 1838), and the dispersion relation of waves (in Lamb's textbook and elsewhere) are all known.Breaking and run-up are not treated——only linear waves whose height is small compared with the depth are considered; the height reached on land is not stated.Actual tsunami events are not treated——arrival times and recorded heights are not discussed; depths and periods are only set and computed.The cause of the discrepancy on the shelf has not been checked——part of the wave may be reflected at the kink at the end of the slope, but this is kept as a hypothesis.Relation to earlier papers**: **Paper 391 showed that the significant wave height is neither the mean nor the maximum, and left open in its scope note that "breaking and shoaling are not the subject"——this paper treats, within shoaling, the growth of a tsunami's height.**What is added is **setting tsunamis and wind waves side by side through numerical solution of the dispersion relation**, **matching Green's law with a numerical solution of the linear shallow-water equations, showing in numbers where it fits and where it departs**, and **placing the separator on whether the wavelength is longer than the depth**.

1. Notation and the Question

Setting: **let h be the depth, $g = 9.80665$ m/s^2 the acceleration of gravity, T the wave period and k the wavenumber (2π divided by the wavelength).The speed of a linear wave is fixed by the dispersion relation**

$$\left(\frac{2\pi}{T}\right)^2 = g\,k\tanh(kh).$$

When kh is small (wavelength much longer than the depth) the speed is $\sqrt{gh}$; when kh is large (shorter), it is $gT/(2\pi)$.

Question: **what fixes the speed of a tsunami, and how much does its height grow in shallow water?**

2. Speed

Computation 2.1: **the speed $\sqrt{gh}$ of waves much longer than the depth was found.**

depth	speed	per hour
4000 m	**198.057062** m/s	**713.005425** km
1000 m	**99.028531** m/s	**356.502712** km
100 m	**31.315571** m/s	**112.736056** km
10 m	**9.902853** m/s	**35.650271** km

Crossing 10000 km of ocean 4000 m deep takes 14.025139 hours.

Computation 2.2: **the dispersion relation was solved numerically (a route independent of the $\sqrt{gh}$ approximation).**

wave (depth)	wavelength	kh	speed over $\sqrt{gh}$
10-minute tsunami (4000 m)	117.947696 km	0.213084	**0.992540**
20-minute tsunami (4000 m)	237.225441 km	0.105945	0.998136
10-second wind wave (4000 m)	156.078 m	161.027130	0.078804
10-second wind wave (10 m)	92.356 m	0.680324	0.932618

The speed 15.607768 m/s of the 10-second wind wave at 4000 m matched the deep-water formula $gT/(2\pi)$.

> **Observation 2.3**
>
> **a tsunami's speed is fixed by the depth alone, a wind wave's by the period alone.A 10-minute tsunami has a wavelength nearly 30 times the depth of 4000 m and moves the water down to the sea floor——in the same sea, a 10-second wave has a wavelength of 156 m and does not feel the floor 4000 m below.**

3. Growth of the Height

> **Observation 3.1**
>
> **this is the core: as the water shallows, the height grows as the inverse $1/4$ power of the depth.The energy a wave carries per period is proportional to the square of its height, and it is carried at the speed $\sqrt{gh}$. If the depth changes slowly this flow is kept, so**

$$a^2\sqrt{h} = \text{const}, \qquad a \propto h^{-1/4}$$

(Green's law).

Computation 3.2: **taking the height at 4000 m as 1, the ratios of height and wavelength were found.**

depth	height ratio	wavelength ratio
1000 m	1.414214	0.500000
400 m	1.778279	0.316228
100 m	**2.514867**	0.158114
40 m	3.162278	0.100000
10 m	4.472136	0.050000

4. Numerical Computation

Computation 4.1: **independently of Green's law, the linear shallow-water equations were solved numerically.A flat sea 4000 m deep for 300 km shoaled uniformly to 40 m over 600 km, followed by a 300-km shelf at 40 m. With a grid spacing of 200 m (6000 points), a hump 1 m high and 20 km wide was sent to the right, and the maximum height at each point was recorded.**

depth	numerical ratio	Green's law	relative difference
4000 m	1.000000	1	—
1000 m	1.422233	1.414214	+0.005671
400 m	1.791855	1.778279	+0.007634
100 m	2.555027	2.514867	+0.015969
40 m (shelf)	3.101584	3.162278	−0.019193

> **Observation 4.2**
>
> **on the slope, the numerical computation agreed with Green's law to within 1.6 per cent.The difference widens with shallower water, +0.015969 at 100 m——Green's law assumes the depth changes slowly, and as the wavelength shrinks the slope becomes relatively steeper, departing from that assumption.**

Observation 4.3

on the shelf beyond the slope, the numerical computation fell below Green's law.At the 40-m shelf, 3.101584 against 3.162278, a relative difference of −0.019193——the cause has not been checked. Part of the wave may be reflected at the kink at the end of the slope, but this paper keeps it as a hypothesis.

5. The Separator

Observation 5.1

the separator is whether the wavelength is longer than the depth (whether kh is small).A tsunami, far longer than the depth, has a speed fixed by the depth alone, and slows and grows taller as the water shallows——a short wind wave in the deep sea has a speed fixed by its period alone and does not feel the floor. Only when it reaches shallow water and kh becomes small (0.680324 at 10 m) does it feel the floor and slow down.

Observation 5.2

beside Paper 391.Paper 391 showed that the significant wave height is neither the mean nor the maximum and, in its scope note, left breaking and shoaling out of its subject——this paper treats, within shoaling, the rate at which a tsunami grows taller. The wind waves of Paper 391 are on the side that does not feel the floor in the deep sea.

Observation 5.3

what is not said.Breaking, run-up and the height reached on land are not treated, nor are the arrival times or recorded heights of actual tsunamis——what is shown is that the speed is fixed by the depth, set beside the dispersion relation, and that the growth of the height, matched between formula and numerical computation, fits in some places and departs in others, in numbers.

6. Roll Call and Closing

1. **198.057062 m/s at 4000 m** (Computation 2.1): **713 km/h.**
2. **10000 km in 14.025139 hours** (Computation 2.1): **crossing the deep ocean.**

3. **a 10-minute tsunami at 0.992540 times $\sqrt{gh}$** (Computation 2.2): **wavelength 118 km.**
4. **wind waves at $gT/(2\pi)$ regardless of depth** (Computation 2.2): **wavelength 156 m.**
5. **height as the inverse $1/4$ power of depth** (Observation 3.1): **the core.**
6. **2.514867 times from 4000 m to 100 m** (Computation 3.2): **wavelength 0.158114 times.**
7. **numerical computation within 0.016 on the slope** (Computation 4.1): **two routes.**
8. **the difference widens in shallower water** (Observation 4.2): **leaving the slow-change assumption.**
9. **0.019193 below on the shelf** (Observation 4.3): **cause not checked.**
10. **the separator is whether the wavelength exceeds the depth** (Observation 5.1): **the seat left open by Paper 391.**

Closing: **A tsunami is a wave far longer than the depth.Its speed is fixed by the depth alone and falls as the water shallows——198.057062 m/s at 4000 m and 9.902853 m/s at 10 m. Solving the dispersion relation, a 10-minute tsunami travels at 0.992540 times $\sqrt{gh}$, while a 10-second wind wave's speed is fixed by its period regardless of depth.As the water shallows, the height grows as the inverse $1/4$ power of the depth——because the flow of energy carried is kept, the height grows 2.514867 times from 4000 m to 100 m. A numerical solution of the linear shallow-water equations agreed with this on the slope to within 1.6 per cent and fell 0.019193 below it on the shelf beyond.The separator is whether the wavelength is longer than the depth——if longer, the wave feels the depth, slows and grows taller; if shorter, it does not feel the floor of the deep sea.Placed among the earlier papers——it takes the seat of shoaling left open in the scope note of Paper 391.To be honest——breaking, run-up and actual events are not treated, and the cause of the shortfall on the shelf has not been checked.**

References

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Yuuki YAMAGISHI (山岸 勇輝)

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