A Siphon’s Flow Is Set by the Drop to the Outlet, and Its Limit by the Height of the Crest Above the Outlet

— The speed of water through a siphon is fixed only by the drop from the upper surface to the outlet; the height of the crest does not enter — the pressure at the crest is fixed only by the height of the crest above the outlet; the level of the upper surface does not enter — [Paper 620]

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— *From thinking to syncing.* —

Abstract

A siphon is a pipe that carries water from an upper tank over a crest higher than the water surface down to a lower outlet.Which height fixes the flow speed and which fixes the crest pressure are found by two routes.No new theorem or law is claimed. First, the flow speed is fixed by the drop from the upper surface to the outlet——without friction the speed is $\sqrt{2g\Delta h}$: 3.131557 m/s for a drop of 0.5 m, 4.428691 m/s for 1 m and 11.717214 m/s for 7 m. However high the crest, it does not enter this formula (Section 2). **Second, and this is the core.The crest pressure is fixed only by the height of the crest above the outlet——the crest pressure found from the upper surface, subtracting the part due to speed, and found from the atmospheric pressure at the outlet, subtracting the hydrostatic part, agreed for all four drops. With the crest 3 m above the surface, it is 62.169008 kPa for an outlet 1 m below and 3.435020 kPa for 7 m below** (Section 2). **Third, as the upper tank empties, the crest pressure does not change——computing the fall of the upper surface with the outlet height fixed, the flow slows, but the crest pressure stays at 62.169008 kPa with differences below 3×10^{-11} Pa. The 350.62 s taken for the surface to fall to 0.05 m above the outlet matched Torricelli’s formula to a relative difference of 7×10^{-7}** (Section 3). **Fourth, the limit is a crest about 10 m above the outlet——when the crest pressure falls to the saturated vapour pressure of water, the water breaks. That height difference is 10.111964 m at 20 °C, 9.182916 m at 50 °C and 5.656917 m at 80 °C, lower for warmer water** (Section 4). **Fifth, the separator is which height is measured——the flow speed looks from the surface to the outlet, the crest pressure from the crest to the outlet. The remaining height enters neither formula** (Section 5). **Keywords**: siphon, Bernoulli’s theorem, saturated vapour pressure, pressure at the crest, head difference.

Scope of this paper (scope note): **No new theorem or law is claimed——Bernoulli's theorem linking speed and pressure (Bernoulli, 1738), the height limit of a siphon (Boatwright, Hughes and Barry, 2015, and others), and tables of the saturated vapour pressure and density of water (the formulation of Wagner and Pruß) are all known.The dispute over how siphons work is not entered——whether the water is "pulled" or "pushed" is not stated; only numbers for pressure and speed are given.Friction and bubbles are not included——pipe friction, bubbles of dissolved air and changes in pipe diameter are not treated.Water properties are representative table values——densities and saturated vapour pressures at 20, 50 and 80 °C are used.Relation to earlier papers**: **Paper 366 showed that an hourglass keeps time because the remaining height is not in its flow formula——in a siphon, the height of the crest is not in the formula for the flow speed, and the height of the upper surface is not in the formula for the crest pressure.**What is added is **finding the crest pressure by two routes, from the upper surface and from the outlet, and confirming they agree**, **showing by a computation in time that the crest pressure does not change as the tank empties**, and **giving the limit for each water temperature and placing the separator on which height is measured**.

1. Notation and the Question

Setting: **taking the upper tank's surface as the reference, let the crest of the siphon be at height z_c and the outlet at z_o (negative if below the surface).Let ρ be the density of water, $p_a = 101325$ Pa the atmospheric pressure and $g = 9.80665$ m/s^2, with no pipe friction.The speed of the jet leaving the outlet, with drop $\Delta h = -z_o$ from the surface, is**

$$v = \sqrt{2g\Delta h}.$$

Question: **what fixes the crest pressure, and where is the limit for unbroken flow?**

2. Speed and Crest Pressure

Computation 2.1: **at 20 °C (density 998.2 kg/m^3), with the crest 3 m above the surface and the outlet drop varied, the crest pressure was found by two routes.(i) From the surface: $p_c = p_a - \rho g z_c - \rho v^2/2$. (ii) From the outlet: the outlet is at atmospheric pressure and the water moves at the same speed throughout the pipe, so $p_c = p_a - \rho g(z_c - z_o)$.**

outlet drop	speed	crest pressure (from surface)	crest pressure (from outlet)
0.5 m	3.131557 m/s	67.063507 kPa	67.063507 kPa
1.0 m	4.428691 m/s	**62.169008** kPa	62.169008 kPa
3.0 m	7.670717 m/s	42.591012 kPa	42.591012 kPa
7.0 m	11.717214 m/s	3.435020 kPa	3.435020 kPa

> **Observation 2.2**
>
> this is the core: the crest pressure is fixed only by the height of the crest above the outlet.Seen from the surface, two parts are subtracted from the crest pressure, one for the crest's height and one for the flow speed. The speed part $\rho v^2/2$ is exactly $\rho g \Delta h$——added together, what is subtracted is only the height difference $z_c - z_o$ from crest to outlet. The height of the upper surface is left nowhere.

3. As the Tank Empties

Computation 3.1: water was run from an upper tank of cross-section $1\ \mathrm{m}^2$ through a pipe of cross-section $0.001\ \mathrm{m}^2$, with the outlet fixed 1 m below the initial surface, and followed in steps of 0.01 s until the surface fell to 0.05 m above the outlet (crest 3 m above the initial surface).It took 350.62 s, matching the 350.619757 s from Torricelli's formula to a relative difference of 6.9×10^{-7}. Throughout, the crest pressure was 62.169008 kPa at both its maximum and minimum, differing by 2.9×10^{-11} Pa.

> **Observation 3.2**
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> the crest pressure does not change as the upper surface falls.As the surface falls, the crest stands higher above it and the flow slows. The greater height lowers the pressure and the slower flow raises it——the two cancel exactly, leaving only the height of the crest above the outlet.

4. The Limit

Computation 4.1: when the crest pressure falls to the saturated vapour pressure of water, the water boils and breaks. The limiting height difference $(p_a - p_v)/(\rho g)$ was found for each water temperature.

water temperature	density	saturated vapour pressure	limit of crest above outlet
20 °C	998.2 kg/m³	2.339 kPa	10.111964 m
50 °C	988.0 kg/m³	12.352 kPa	9.182916 m
80 °C	971.8 kg/m³	47.414 kPa	5.656917 m

Without vapour pressure, it would be 10.350906 m at 20 °C.

> **Observation 4.2**
>
> the limit is a crest about 10 m above the outlet.In Computation 2.1 with the outlet 7 m below, the crest is 10 m above the outlet, and its pressure of 3.435020 kPa is just above the 2.339 kPa vapour pressure at 20 °C——lowering the outlet a little further would speed the flow but break the water at the crest. Warmer water has a higher vapour pressure, and at 80 °C the limit falls to 5.656917 m.

5. The Separator

> **Observation 5.1**
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> the separator is which height is measured.The flow speed is fixed by the drop from the upper surface to the outlet, without the crest's height. The crest pressure and the limit are fixed by the height of the crest above the outlet, without the upper surface's height——the same siphon has three heights, and the two questions each pick a different pair.

> **Observation 5.2**
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> beside Paper 366.Paper 366 showed that water's flow changes with the remaining height, but sand's flow shows no height, which is why an hourglass keeps time——a siphon's flow speed changes with the drop, as water does. But the height of the upper surface does not appear in the formula for the crest pressure. The height missing from a formula decides what stays constant.

Observation 5.3

what is not said.The dispute over whether a siphon “pulls” or “pushes” the water is not entered. Pipe friction, dissolved-air bubbles and changes in pipe diameter are not included——what is shown is that the crest pressure found by two routes agrees, that it does not change as the tank empties, and that the limit falls with water temperature.

6. Roll Call and Closing

1. **flow speed $\sqrt{2g\Delta h}$** (Notation): **the crest height does not enter.**
2. **4.428691 m/s for a 1-m drop** (Computation 2.1): **11.717214 m/s for 7 m.**
3. **crest pressure agrees by two routes** (Computation 2.1): **for all four drops.**
4. **fixed only by the crest’s height above the outlet** (Observation 2.2): **the core.**
5. **62.169008 kPa while the tank empties** (Computation 3.1): **differences below 3×10^{-11} Pa.**
6. **time matches Torricelli to 7×10^{-7}** (Computation 3.1): **350.62 s.**
7. **height and slowness cancel** (Observation 3.2): **the upper surface is left nowhere.**
8. **limit at 20 °C: 10.111964 m** (Computation 4.1): **crest above outlet.**
9. **5.656917 m at 80 °C** (Computation 4.1): **vapour pressure rises.**
10. **the separator is which height is measured** (Observation 5.1): **two of three heights.**

Closing: **A siphon carries water over a crest higher than the water surface.Its flow speed is fixed by the drop from the upper surface to the outlet; the crest’s height does not enter——4.428691 m/s for a 1-m drop and 11.717214 m/s for 7 m.The crest pressure is fixed only by the crest’s height above the outlet; the upper surface does not enter——found from the upper surface or from the outlet it gives the same value, and it stayed at 62.169008 kPa as the tank emptied. The greater height of the crest and the slower flow cancel.The limit is a crest about 10 m above the outlet——10.111964 m at 20 °C and 5.656917 m at 80 °C; beyond that, the water breaks at the crest.Placed among the earlier papers——as with the hourglass of Paper 366, the height missing from a formula decides what stays constant.To be honest——friction and bubbles are not included, and the dispute over how siphons work is not entered.**

References

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