The Look-and-Say Sequence Grows by About 1.3036 Each Step, Whatever the Seed

— Read 1 as “one 1” to get 11, read that as “two 1s” to get 21, then 1211 — only the seed 22 never moves —

[Paper 623]

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— *From thinking to syncing.* —

Abstract

Consider the sequence whose next term is made by reading the previous one aloud as “how many of which digit”. Starting from 1 it runs 1, 11, 21, 1211, 111221, . . . (the look-and-say sequence).How its length grows is counted for several seeds (first terms).No new theorem or law is claimed. First, the length grows by about 1.3036 each step——from the seed 1, term 10 has 26 digits, term 30 has 5808 and term 51 has 1520986. The ratio averaged over ten terms (the length of term 51 divided by that of term 41, to the power 1/10) is 1.303610 (Section 2). **Second, and this is the core.The ratio does not depend on the seed——changing the seed to 3, 123 or 1113, the ratio averaged over ten terms is 1.303747, 1.303682 and 1.303475, all within 0.000170 of Conway’s constant 1.303577. The lengths themselves differ by seed (from 1520986 to 4070770 at term 51), but the growth is the same** (Section 3). **Third, the ratio from one term to the next wobbles——the ratio of term 51 to term 50 ranges over the four seeds from 1.302897 to 1.303918, a spread of 0.001021. Averaging over ten terms shrinks the spread to 0.000272** (Section 3). **Fourth, only the seed 22 never moves——22 reads as “two 2s” and returns to 22; its length stays 2 for every term** (Section 4). **Fifth, from the seed 1 only 1, 2 and 3 appear——checking up to term 40, no digit 4 or higher ever appeared, because four equal digits in a row never occur** (Section 4). **Keywords**: look-and-say sequence, Conway’s constant, growth rate, splitting theorem, integer sequence.

> **Scope of this paper (scope note)**: **No new theorem or law is claimed——that the length of the look-and-say sequence grows at the same constant whatever the seed (Conway, 1987) and the proof of the decomposition theorem behind it (Ekhad and Zeilberger, 1997) are known.Where the constant comes from is not treated——Conway's constant is known as a root of a polynomial of degree 71, but neither that polynomial nor the division of the sequence into 92 "elements" is treated here; only the known value 1.303577 is compared with counted lengths.Relation to earlier papers**: **Paper 161 showed that "fixed point" theorems are different things according to what each demands and delivers——the seed 22 is the fixed point of reading aloud, and no other seed goes there; they all keep growing at the same rate.Paper 622 showed that the repeated sorting and subtracting of digits stops or circles depending on the number of digits——repeated reading aloud never stops, and only the growth is shared regardless of the seed.**What is added is **counting the lengths up to term 51 for four seeds**, **setting out in numbers the wobble of the term-to-term ratio and how the ten-term ratio lines up**, and **placing the separator on whether the seed is 22**.

1. Notation and the Question

Setting: **reading a string of digits from the left, each run of equal digits is written out as "count, digit" to form the next term. Let L_n be the number of digits of term n (the seed being term 0).**

Question: **how does the length grow, and what changes and what does not when the seed changes?**

2. Counting Lengths

Computation 2.1: **for five seeds, terms up to 51 were made and their lengths counted.**

seed	term 10	term 30	term 51
1	26	5808	1520986
3	30	6784	1766402
123	74	15570	4070770
1113	56	11420	3002354
22	2	2	2

Observation 2.2

the length depends on the seed.At term 51 the seed 123 is about 2.7 times as long as the seed 1——whether every seed grows in the same way must be seen in the ratio, not the length itself.

3. The Ratio

Computation 3.1: **the term-to-term ratio L_{51}/L_{50} and the ratio averaged over ten terms, $(L_{51}/L_{41})^{1/10}$, were found.**

seed	L_{51}/L_{50}	$(L_{51}/L_{41})^{1/10}$
1	1.303730	**1.303610**
3	1.303089	1.303747
123	1.303918	1.303682
1113	1.302897	1.303475

The spread (largest minus smallest) of the term-to-term ratio was 0.001021, and that of the ten-term ratio 0.000272. All four ten-term ratios were within 0.000170 of Conway's constant 1.303577.

Observation 3.2

this is the core: the ratio does not depend on the seed.Changing the seed among 1, 3, 123 and 1113 changes the length by up to about 2.7 times, but the ten-term ratios all line up at the same constant——the seed decides the starting length, not the growth.

Observation 3.3

the term-to-term ratio wobbles.Reading aloud depends on the fine shape of the digit string, so the ratio of one term to the next moves up and down by about 0.001——averaged over ten terms, the wobble shrinks to about a quarter and the constant comes into view.

4. The Separator

Observation 4.1

the separator is whether the seed is 22.22 reads as “two 2s” and returns to 22; its length stays 2 for every term——every other seed keeps growing at the same rate. Only one seed stands still.

Computation 4.2: **checking the digits appearing up to term 40 from the seed 1, only 1, 2 and 3 occurred.**

Observation 4.3

digits of 4 or more do not appear unless the seed has them.Reading aloud writes a 4 only when four equal digits are in a row, which does not happen starting from 1——the limited set of digits also supports the growth being independent of the seed.

Observation 4.4

beside Papers 161 and 622.Paper 161 showed that fixed-point theorems differ by what each demands and delivers, and Paper 622 that repeatedly sorting and subtracting digits stops or circles depending on the number of digits——in repeated reading aloud, the fixed point 22 exists but attracts no other seed. The repetition never stops, and only the growth lines up.

Observation 4.5

what is not said.Where Conway’s constant comes from (the polynomial of degree 71, the 92 elements) is not treated——what is shown, from lengths counted for four seeds, is that the ratio lines up at the same constant, that it wobbles from term to term, and that only the seed 22 does not move.

5. Roll Call and Closing

1. **term 51 from the seed 1 has 1520986 digits** (Computation 2.1): **about 1.3036 times per step.**
2. **lengths differ by up to about 2.7 times by seed** (Observation 2.2): **different starts.**
3. **ten-term ratio 1.303610 (seed 1)** (Computation 3.1): **close to the constant.**

4. **all four seeds within 0.000170 of the constant** (Computation 3.1): **two routes.**
5. **the ratio does not depend on the seed** (Observation 3.2): **the core.**
6. **the term-to-term ratio spreads by 0.001021** (Observation 3.3): **0.000272 over ten terms.**
7. **the seed 22 never moves** (Observation 4.1): **the separator.**
8. **only 1, 2, 3 from the seed 1** (Computation 4.2): **up to term 40.**
9. **four equal digits never occur** (Observation 4.3): **a limited set of digits.**
10. **a fixed point that attracts nothing** (Observation 4.4): **Papers 161 and 622.**

Closing: **Making each term by reading the previous one aloud, the length grows by about 1.3036 each step.That ratio does not depend on the seed——changing the seed among 1, 3, 123 and 1113 changes the length by up to about 2.7 times, but the ten-term ratios all lined up within 0.000170 of Conway's constant 1.303577. The term-to-term ratio wobbles by about 0.001.The separator is whether the seed is 22——only 22 returns to itself and stands still; every other seed keeps growing at the same rate.Placed among the earlier papers——beside the fixed points of Paper 161 and the digit repetition of Paper 622, here a fixed point exists yet attracts nothing, and only the growth lines up.To be honest——where the constant comes from is not entered; lengths were counted and compared with the known value.**

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