The Number of Ways to Place Non-Attacking Queens Does Not Always Grow with the Board

— Place n queens on an $n \times n$ board so that none attacks another along a row, column or diagonal — 12×12 has 14200 — [Paper 624]

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— *From thinking to syncing.* —

Abstract

A chess queen moves any distance along rows, columns and diagonals. The ways to place n queens on an $n \times n$ board with no two attacking each other are counted (the n-queens problem).No new theorem or law is claimed. First, the 8×8 board has 92 placements, 12 when merged——merging placements that coincide under the 8 rotations and reflections of the board, the 92 become 12 (Section 2). **Second, small boards have none——on 2×2 and 3×3 any placement attacks. The first placements appear on 4×4: 2 (1 when merged)** (Section 2). **Third, and this is the core.A larger board does not always have more placements——5×5 has 10, but 6×6 drops to 4 (1 when merged). 7×7 comes back to 40, and 12×12 rises to 14200** (Section 2). **Fourth, two searches agree——a search placing one row at a time while recording used columns and both diagonals in bits, and a search trying all $n!$ column orders, gave the same counts for $n = 4$ to 8** (Section 3). **Keywords**: eight queens puzzle, n-queens problem, backtracking, board symmetry, combinatorial search.

Scope of this paper (scope note): **No new theorem or law is claimed——the 92 placements of eight queens and the counts for each n are known (the survey by Bell and Stevens of 2009, the paper by Rivin and coauthors of 1994, and others).Asymptotics for general n are not treated——$n = 1$ to 12 are counted, with symmetries merged for $n \leq 10$ .Fast construction is not treated——to construct a single placement, methods without counting them all are known.Relation to earlier papers**: **Paper 604 showed that no Latin square of order six has an orthogonal mate——Latin squares and queen placements are both arrangements with one per row and column; queens add the diagonal condition. Here too the count dips at 6.**What is added is **setting out the counts for $n = 1$ to 12 and the merged counts for $n \leq 10$, matching two searches for $n = 4$ to 8**, and **placing the separator on the board size and showing that the growth is not monotone**.

1. Notation and the Question

Setting: place n queens on an $n \times n$ board so that no two share a row, a column or a diagonal line.Exactly one queen goes in each row, so the placements are among the column orders ($n!$ of them).

Question: how many placements are there, and does a larger board have more?

2. Counting

Computation 2.1: counted by a search that places one row at a time, recording used columns and both diagonals in bits and trying only safe squares.Merging placements that coincide under the 8 rotations and reflections was also counted for $n \leq 10$.

n	1	2	3	4	5	6	7	8	9	10	11	12
placements	1	0	0	2	10	4	40	92	352	724	2680	14200
merged	1	0	0	1	2	1	6	12	46	92	—	—

> **Observation 2.2**
>
> the 92 placements of 8×8 become 12 when merged.Placements carried into each other by rotation and reflection can be regarded as the same shape——there are only 12 genuinely different shapes.

> **Observation 2.3**
>
> this is the core: a larger board does not always have more placements.From 10 on 5×5, 6×6 drops to 4; merged, 6×6 has only 1——7×7 comes back to 40 and from there the counts grow. The 0 of 2×2 and 3×3 and the dip at 6×6 are set by fine properties of the board size.

3. Two Searches

Computation 3.1: independently of the bit search, all $n!$ column orders were generated and those without diagonal clashes counted.For $n = 4$ to 8 both searches gave the same counts $(2, 10, 4, 40, 92)$.

Observation 3.2

of the arrangements with one per row and column, only a handful survive the diagonals.On 8×8, only 92 of the $8! = 40320$ orders remain——the diagonal condition removes the great majority.

4. The Separator

Observation 4.1

the separator is the board size n itself.The count tends to grow with n but dips once at $n = 6$. It cannot be said that a larger n always has more——not only the size but how the diagonal lines cut the board decides the count.

Observation 4.2

beside Paper 604.Paper 604 showed that no Latin square of order six has an orthogonal mate——Latin squares and queen placements are both arrangements with one per row and column, and both dip at 6. But queens can still be placed in 4 ways on 6×6, so they do not disappear.

Observation 4.3

what is not said.Asymptotics for general n and fast methods to construct one placement are not treated—— what is shown is the count up to $n = 12$, the merged counts, and the agreement of two searches.

5. Roll Call and Closing

1. **92 placements on 8×8** (Computation 2.1): **agreeing with the known value.**
2. **12 when merged** (Observation 2.2): **by rotation and reflection.**
3. **0 on 2×2 and 3×3** (Computation 2.1): **no placement.**
4. **10 on 5×5, 4 on 6×6** (Observation 2.3): **the core.**
5. **6×6 has 1 when merged** (Computation 2.1): **a single shape.**
6. **14200 on 12×12** (Computation 2.1): **growing with size overall.**

7. **two searches agree for $n = 4$ to 8** (Computation 3.1): **two routes.**
8. **92 of 40320** (Observation 3.2): **diagonals remove most.**
9. **the separator is the board size itself** (Observation 4.1): **not monotone.**
10. **a dip at 6, as with Latin squares** (Observation 4.2): **Paper 604.**

Closing: **Place n non-attacking queens on an $n \times n$ board.On 8×8 there are 92 placements, 12 when merged by rotation and reflection——two searches agreed for $n = 4$ to 8.A larger board does not always have more placements ——2×2 and 3×3 have none, and the 4 on 6×6 are fewer than the 10 on 5×5. From 7×7 on the counts grow, reaching 14200 on 12×12.Placed among the earlier papers——like the Latin squares of Paper 604 these are arrangements with one per row and column, and both dip at 6.To be honest——only boards up to 12 were counted; the behaviour for general n is not entered.**

References

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