Eight Perfect Shuffles Restore 52 Cards, While Seven Random Riffles Nearly Mix Them

— keeping the top card on top restores the order in **8** shuffles, putting it second takes **52** — done at random, the same interleaving brings the distance from uniform (total variation) down to **0.334061** after **7** riffles — [Paper 626]

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— From thinking to syncing. —

Abstract

Splitting a deck in half and interleaving one card at a time from each half is a perfect shuffle. One that keeps the top card on top is an out-shuffle; one that makes the top card second is an in-shuffle.How many shuffles restore the order, and how many random riffles of the same kind mix it, are counted.No new theorem or law is claimed. First, 52 cards return after 8 out-shuffles or 52 in-shuffles——shuffling and counting, 8 out-shuffles and 52 in-shuffles restored the order (Section 2). **Second, the count is the order of 2——the number of out-shuffles is how many powers of 2 it takes for the remainder on division by $n-1$ first to be 1; for in-shuffles, on division by $n+1$. For every even deck from 4 to 100 cards it matched the actual shuffling. 50 cards need 21 out-shuffles and 8 in-shuffles, 64 cards 6 and 12: one card more or less changes the count greatly** (Section 2). **Third, the top card can be sent anywhere——writing the target position in binary and doing an in-shuffle for each 1 and an out-shuffle for each 0 from the top digit, the top card was sent to every position 0 to 51 of 52 cards within 6 shuffles** (Section 3). **Fourth, and this is the core.The same interleaving restores the order when perfect and mixes it when random——in a riffle with a random cut and random drops (the Gilbert--Shannon--Reeds model), the distance from uniform (total variation) stays 1.000000 up to 4 riffles, then falls to 0.923733 at 5, 0.613550 at 6, 0.334061 at 7 and 0.167159 at 8. In the same number that the perfect shuffle restores the order, random riffles nearly finish mixing it** (Section 4). **Keywords**: perfect shuffle, faro shuffle, multiplicative order of 2, riffle shuffle, card mixing.

> **Scope of this paper (scope note)**: **No new theorem or law is claimed——that the number of perfect shuffles is set by the order of 2 (the paper by Diaconis, Graham and Kantor of 1983 and others) and the formula for how random riffles mix (Bayer and Diaconis, 1992) are known.Card-trick technique is not discussed——whether a perfect shuffle can be done by hand is not treated.One model of random riffles is used——whether human shuffling follows this model is not treated.Relation to earlier papers**: **Paper 603 followed the remainders of powers of 2 on division by 561 to examine Fermat's test——here the number of powers of 2 before the remainder on division by 51 returns to 1 is the number of shuffles before the cards return.**What is added is **matching the route of actual shuffling with the order of 2 for decks of 4 to 100 cards**, **confirming the binary procedure for sending the top card over all 52 positions**, and **placing the separator on fixed versus random shuffling and setting out the distances for random riffles**.

1. Notation and the Question

Setting: **a deck of n cards (n even) is split into upper and lower halves and interleaved one card at a time. If the first card of the upper half ends on top it is an out-shuffle; if the first card of the lower half does, an in-shuffle.In an out-shuffle the top and bottom cards stay, and every other card moves from position i to $2i$ modulo $n-1$; in an in-shuffle a card moves from position i (counting from 1) to $2i$ modulo $n+1$.**

Question: **how many shuffles restore the order, and how many random versions of the same shuffle mix the deck?**

2. Shuffles to Return

Computation 2.1: **the number of shuffles until the order returned was compared with the number of powers of 2 until the remainder first equals 1 (the order of 2).**

cards	out (order of 2 mod $n-1$)	in (order of 2 mod $n+1$)
8	3	6
10	6	10
32	5	10
50	21	8
52	8	52
54	52	20
64	6	12

For every even deck from 4 to 100 cards, the actual shuffles and the order of 2 agreed.

> **Observation 2.2**
>
> 52 cards need only 8 out-shuffles because $2^8 = 256$ leaves remainder 1 on division by 51.An in-shuffle divides by 53, where the remainder first returns to 1 at 2^{52}, so it takes 52——keeping the top card on top or putting it second divides by 51 or 53, and splits the counts into 8 and 52.

3. Sending the Top Card

Computation 3.1: the target position p (0 to 51) was written in binary, and from the top digit an in-shuffle was done for each 1 and an out-shuffle for each 0.In all 52 cases the top card arrived at position p. Since 51 has 6 binary digits, at most 6 shuffles are needed.

> **Observation 3.2**
>
> an in-shuffle doubles the top card's position and adds 1; an out-shuffle doubles it.So it is the same as writing binary digits one at a time from the top——a perfect shuffle is merely doing binary arithmetic with cards.

4. Random Riffles

Computation 4.1: **for a random riffle with a binomial cut and each card dropped from a half with probability proportional to its remaining size (the Gilbert--Shannon--Reeds model), the distance of 52 cards from uniform after k riffles (total variation) was computed exactly by the formula of Bayer and Diaconis.**

riffles	4	5	6	7	8	9	10	12
total variation	1.000000	0.923733	0.613550	**0.334061**	0.167159	0.085420	0.042946	0.010755

> **Observation 4.2**
>
> **this is the core: the same interleaving restores the order when perfect and mixes it when random.Perfect shuffles restore 52 cards in 8. Random riffles keep traces of the original order up to 4 (distance 1.000000), and mix to 0.334061 at 7 and 0.167159 at 8——what separates them is not the number of shuffles but whether the shuffle is fixed or random.**

5. The Separator

> **Observation 5.1**
>
> **the separator is whether the shuffle is fixed or random.If fixed, the rearrangement repeats one permutation and must eventually return. If random, the rearrangement differs each time and the order approaches uniform——and even among fixed shuffles, out versus in changes the divisor and the count from 8 to 52.**

> **Observation 5.2**
>
> **beside Paper 603.Paper 603 followed the remainders of powers of 2 on division by 561 and showed that 561 passes the test though not prime——here the same remainders of powers of 2 count the shuffles until the cards return. 53 is prime and the order of 2 reaches the maximum 52, so in-shuffles take 52.**

Observation 5.3

what is not said.Card-trick technique and whether human shuffling follows the random-riffle model are not treated——what is shown is the agreement of actual shuffling with the order of 2, the binary procedure for sending cards, and the distances for random riffles.

6. Roll Call and Closing

1. **52 cards return in 8 out-shuffles** (Computation 2.1): 2^8 **leaves 1 modulo 51.**
2. **52 in-shuffles** (Computation 2.1): **because it divides by 53.**
3. **agreement with the order of 2 for 4 to 100 cards** (Computation 2.1): **two routes.**
4. **50 cards need 21 out-shuffles** (Computation 2.1): **the count changes greatly with size.**
5. **binary sends the top card anywhere** (Computation 3.1): **within 6 shuffles.**
6. **in doubles and adds 1, out doubles** (Observation 3.2): **writing binary digits.**
7. **random riffles reach 0.334061 at 7** (Computation 4.1): **nearly mixed.**
8. **1.000000 up to 4** (Computation 4.1): **traces remain.**
9. **perfect returns, random mixes** (Observation 4.2): **the core.**
10. **remainders of powers of 2 count the shuffles** (Observation 5.2): **Paper 603.**

Closing: **Splitting the deck in half and interleaving, 52 cards return to their order after 8 out-shuffles or 52 in-shuffles.The count is the order of 2——the number of powers of 2 before the remainder on division by 51 or 53 returns to 1, matching the actual shuffles for every deck from 4 to 100 cards. The top card can be sent anywhere by choosing in and out by its binary digits.The same shuffle returns the order when perfect and mixes it when random ——random riffles bring the distance from uniform to 0.334061 at 7 and 0.167159 at 8.The separator is whether the shuffle is fixed or random.Placed among the earlier papers——the remainders of powers of 2 from Paper 603 here count how often the cards return.To be honest——neither card tricks nor whether people shuffle like the model are entered.**

References

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