

The Velocity–Spacetime Curvature Duality

— Energy as the Hinge between $E = 4\pi/|K|$ and $G_{\mu\nu} = \kappa T_{\mu\nu}$ —

Thomas Precession and Gravitational Holonomy

Author: Yuuki YAMAGISHI (山岸 勇輝)

Affiliation: Yamagishi Complexity Torus Lab, Japan

ORCID: 0009-0006-8587-3450 | **Paper 63** | **Zenodo:** 10.5281/zenodo.21606250

Category: Physics / Geometry / Structural Interpretation

Keywords: velocity space, hyperbolic geometry, curvature duality, Thomas precession, geodetic precession, tangent bundle, structural interpretation

— *From thinking to syncing.* —

Positioning of this paper

This paper offers a **structural interpretation** of established physical laws and mathematical theorems. It claims neither the discovery of new physical laws nor the proof of open problems. The algebraic relations presented are transformations of known expressions; the aim is to propose a geometric reading of them. No claim contradicting established theory is made.

ABSTRACT

Paper 60 established that the rest energy in the $m = 4\pi$ representation can be written as $E = 4\pi/|K_v|$, where $K_v = -1/c^2$ is the sectional curvature of the hyperbolic velocity space (Varičák, 1910). The Einstein field equation establishes $G_{\mu\nu} = (8\pi G/c^4) T_{\mu\nu}$, making spacetime curvature proportional to energy-momentum. This paper observes that these two relations operate in **opposite directions**, with energy E serving as a **hinge**: velocity-space curvature *determines* energy ($E \propto 1/|K_v|$), while energy *determines* spacetime curvature ($R \propto T$). Three structural observations follow: (1) composing the two yields an inverse relationship where the intermediate parameter c cancels algebraically (Proposition 3.1); (2) both curvatures are physically measured as holonomies—Thomas precession for velocity space, geodetic/Lense–Thirring precession for spacetime—which are the same mathematical structure in different arenas (Proposition 4.1); (3) both arenas live on the tangent bundle TM of spacetime, with the Riemann curvature governing the base and the hyperbolic curvature governing the fiber, unified by the equivalence principle as fiber constancy (Proposition 5.1). No new prediction is made; the duality is a structural re-organization of known relations. Whether it possesses content beyond re-organization is stated as an explicit open problem.

1. Introduction

Paper 60 showed that the rest energy $E = mc^2$, when m is replaced by 4π (the Yamagishi solid-angle patch), takes the form $E = 4\pi c^2$. Because the velocity space of special relativity is a hyperbolic 3-space of sectional curvature $K_v = -1/c^2$ (Varičák, 1910; cf. Rhodes & Semon, 2004), this is equivalently

$$E = \frac{4\pi}{|K_v|}$$

where $4\pi = \int_{S^2} d\Omega$ is the full solid angle (Paper 01) and $1/|K_v| = c^2$ is the squared curvature radius. The curvature K_v is physically measured as Thomas precession.

Independently, the Einstein field equation (Einstein, 1915)

$$G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

relates the curvature of spacetime (left) to the energy-momentum content (right).

The aim of this paper is to observe that these two relations are **structurally dual**: one says "curvature determines energy," the other says "energy determines curvature." The observation is elementary, but the structural pattern it reveals—energy as a hinge, holonomy as the common measurement, and the tangent bundle as the common habitat—suggests that the two curvatures, though living in different arenas, may not be unrelated.

What this paper does not do. It does not derive, prove, or predict anything beyond what is already known. The hinge structure is a re-organization; whether it has deeper content is an open problem.

2. Review: The Two Curvature Relations

2.1 Velocity-space curvature determines energy (Paper 60)

In special relativity, the set of physically admissible velocities $\{|v| < c\}$ carries a natural Riemannian metric induced by the Lorentz group. Under this metric, velocity space is a 3-dimensional hyperbolic space $\mathbb{H}^3$ of constant sectional curvature $K_v = -1/c^2$ (Varičák, 1910; Ungar, 2001; Rhodes & Semon, 2004). The curvature radius is c .

Under the $m = 4\pi$ patch:

$$E = 4\pi c^2 = \frac{4\pi}{|K_v|}$$

Direction: curvature $\rightarrow$ energy. The curvature K_v is given (by the value of c), and E follows.

2.2 Energy determines spacetime curvature (Einstein, 1915)

The Einstein field equation states that the Einstein tensor $G_{\mu\nu}$ (a measure of spacetime curvature) is proportional to the energy-momentum tensor $T_{\mu\nu}$. Under $G = 4\pi$ (Yamagishi patch, Paper 35):

$$G_{\mu\nu} = \frac{32\pi^2}{c^4} T_{\mu\nu}$$

Direction: energy $\rightarrow$ curvature. The energy-momentum $T_{\mu\nu}$ is given, and the spacetime curvature follows.

3. The Hinge Structure

Proposition 3.1 (Energy as hinge)

The two curvature relations operate in opposite directions:

Arena	Relation	Direction
Velocity space	$E = 4\pi/ K_v $	curvature $\rightarrow$ energy
Spacetime	$G_{\mu\nu} \propto T_{\mu\nu}$	energy $\rightarrow$ curvature

Energy E is the hinge: it is determined by velocity-space curvature and determines spacetime curvature.
Composing the two:

$$K_v \xrightarrow{E=4\pi/|K_v|} E \xrightarrow{G_{\mu\nu} \propto T} R_{\text{spacetime}}$$

In the composition, the parameter c cancels: since $|K_v| = 1/c^2$, we have $E/c^2 = 4\pi/(|K_v| \cdot c^2) = 4\pi/1 = 4\pi$.
The intermediate variable c drops out, leaving only the solid angle 4π .

Caveat (self-limitation). The cancellation $|K_v| \cdot c^2 = 1$ is tautological (since $|K_v| \equiv 1/c^2$). That the intermediate parameter cancels when dual relations are composed is a general algebraic property, not specific to physics. The structural content, if any, lies in the question: *why does the same c appear as both the curvature parameter of velocity space and the conversion factor in the Einstein equation?* This question is not answered here.

4. Two Holonomies

Proposition 4.1 (Both curvatures are measured as holonomies)

Curvature, in differential geometry, is the holonomy: the failure of parallel transport around a closed loop to return to the identity. Both curvatures of this paper are measured as holonomies:

Arena	Holonomy	Measurement
Velocity space	Thomas precession (Wigner rotation)	Spin-orbit coupling ($\frac{1}{2}$ factor) in atomic fine structure (Thomas, 1926)
Spacetime	Geodetic precession (de Sitter)	Gravity Probe B: 6606 ± 18 mas/yr (Everitt et al., 2011)
Spacetime	Lense–Thirring precession (frame dragging)	Gravity Probe B: 37.2 ± 7.2 mas/yr

Thomas precession and geodetic/Lense–Thirring precession are the *same mathematical structure* (parallel-transport deficit around a closed path), realized in *different spaces*.

Corollary 4.2 (Duality of precessions)

Since $E = 4\pi/|K_v|$ with $|K_v|$ measured by Thomas precession, and $R_{\mu\nu}$ measured by geodetic precession, both sides of the hinge (Proposition 3.1) are independently measurable as precession phenomena. The "velocity-space curvature $\leftrightarrow$ spacetime curvature" duality is, experimentally, a "Thomas precession $\leftrightarrow$ geodetic precession" duality.

5. The Tangent Bundle

Proposition 5.1 (Both curvatures live on TM)

The tangent bundle TM of spacetime M is the space of all pairs (x, v) where $x \in M$ is a spacetime event and $v \in T_x M$ is a velocity at x . At each point x , the fiber $T_x M$ contains the velocity space $\{|v| < c\}$. Hence:

- The **base** M carries the Riemann curvature $R_{\mu\nu\rho\sigma}$ (horizontal curvature).
- Each **fiber** carries the hyperbolic curvature $K_v = -1/c^2$ (vertical curvature).

The Einstein equation governs the base. $E = 4\pi/|K_v|$ characterizes the fiber. **Both curvatures live on the same geometric object TM , in complementary (horizontal/vertical) directions.**

Corollary 5.2 (Equivalence principle as fiber constancy)

The equivalence principle guarantees that at every $x \in M$, the tangent space $T_x M$ is Minkowskian. Hence $K_v = -1/c^2$ is **the same at every point** (the locally measured speed of light is always c). The base curvature $R_{\mu\nu\rho\sigma}$ varies (gravity varies from place to place), but the fiber curvature K_v is uniform. This is the **vertical expression of the equivalence principle**.

Note on varying- c theories. In theories where c is position-dependent (Moffat, 1993; Albrecht & Magueijo, 1999), the fiber curvature $K_v(x) = -1/c(x)^2$ would vary over the base. The duality of this paper would then acquire content beyond re-organization: the fiber curvature would become a dynamical quantity coupled to the base. **This paper does not advocate varying- c theories;** it records the observation.

6. Scope of the Claims

What is established

1. $E = 4\pi/|K_v|$ (Paper 60) and the Einstein equation $G_{\mu\nu} = \kappa T_{\mu\nu}$ operate in opposite directions, with energy as the hinge (Proposition 3.1).
2. Both curvatures are measured as holonomies of the same mathematical type (Proposition 4.1, Corollary 4.2).
3. Both live on complementary parts of the tangent bundle TM (Proposition 5.1, Corollary 5.2).

What is not established

- That this duality constitutes a new physical law or produces a new prediction.
- That the cancellation $|K_v| \cdot c^2 = 1$ has content beyond tautology.
- That the tangent-bundle reading modifies any equation of motion.

Open problems

- Does the duality acquire non-trivial content in the **symplectic geometry of phase space** T^*M ? In the ADM formalism of general relativity, the Hamiltonian constraint relates spatial curvature to energy density. If $E = 4\pi/|K_v|$ can be embedded in the ADM constraint, the duality may connect kinematic (velocity-space) and dynamic (spacetime) curvatures within a single Hamiltonian framework.
- In Schwarzschild spacetime, the **coordinate speed of light** varies: $c_{\text{coord}} = c\sqrt{1 - r_s/r}$. Defining $K_{\text{eff}}(r) = -1/c_{\text{coord}}^2$ yields position-dependent fiber curvature. Does $E_{\text{eff}} = 4\pi/|K_{\text{eff}}|$ reproduce the gravitational redshift? If so, is this a re-derivation or a genuine geometric insight?

7. Conclusion

1. $E = 4\pi/|K_v|$ (velocity-space curvature determines energy) and $G_{\mu\nu} = \kappa T_{\mu\nu}$ (energy determines spacetime curvature) are structurally dual, with energy as the hinge. Composing them, the parameter c cancels.
2. Both curvatures are measured as holonomies: Thomas precession (velocity space) and geodetic precession (spacetime).
3. Both live on the tangent bundle TM : spacetime curvature is horizontal, velocity-space curvature is vertical. The equivalence principle is the constancy of the fiber curvature.

This paper claims no new physical law. It observes that two fundamental curvature relations are connected by energy as a hinge, measured by the same mathematical structure (holonomy), and housed in complementary parts of the tangent bundle. Whether this structural duality possesses content beyond re-organization is an open problem.

References

1. Varičák, V. (1910). *Anwendung der Lobatschefschijschen Geometrie in der Relativtheorie*. Phys. Z., 11, 93–96.
 2. Einstein, A. (1915). *Die Feldgleichungen der Gravitation*. Sitzungsber. Preuss. Akad. Wiss., 844–847.
 3. Thomas, L. H. (1926). *The motion of the spinning electron*. Nature, 117, 514.
 4. de Sitter, W. (1916). *On Einstein's theory of gravitation*. Mon. Not. R. Astron. Soc., 77, 155–184.
 5. Everitt, C. W. F. et al. (2011). *Gravity Probe B: Final results*. Phys. Rev. Lett., 106, 221101.
 6. Rhodes, J. A. & Semon, M. D. (2004). *Relativistic velocity space, Wigner rotation, and Thomas precession*. Am. J. Phys., 72, 943–960.
 7. Ungar, A. A. (2001). *Beyond the Einstein Addition Law and its Gyroscopic Thomas Precession*. Kluwer.
 8. Moffat, J. W. (1993). *Superluminary universe*. Int. J. Mod. Phys. D, 2, 351–365.
 9. [Yamagishi 2026, Paper 01] Y. Yamagishi, *The Uniqueness Theorem of $n=2$* . Zenodo. [10.5281/zenodo.21403186](https://zenodo.org/record/105281/1/zenodo.21403186)
 10. [Yamagishi 2026, Paper 35] Y. Yamagishi, *$G = 4\pi$ Defrosting*. Zenodo. [10.5281/zenodo.21430118](https://zenodo.org/record/105281/1/zenodo.21430118)
 11. [Yamagishi 2026, Paper 44] Y. Yamagishi, *The c^1 – c^6 Dimensional Hierarchy*. Zenodo. [10.5281/zenodo.21602231](https://zenodo.org/record/105281/1/zenodo.21602231)
 12. [Yamagishi 2026, Paper 60] Y. Yamagishi, *c as the Radius of Velocity Space*. Zenodo. [10.5281/zenodo.21598012](https://zenodo.org/record/105281/1/zenodo.21598012)
 13. [Yamagishi 2026, Paper 61] Y. Yamagishi, *The Light Sphere $r = ct$* . Zenodo. [10.5281/zenodo.21598127](https://zenodo.org/record/105281/1/zenodo.21598127)
-

— *From thinking to syncing.* —

Yuuki YAMAGISHI (山岸 勇輝)

© 2026 Yamagishi Complexity Torus Lab, Japan · Licensed under Creative Commons Attribution 4.0 International (CC BY 4.0)