The Chance That a Random Chord Is Long Is $1/3$, $1/2$ or $1/4$, Depending on What Is Taken as Uniform

— choosing both ends on the circle gives $1/3$, the midpoint on a radius $1/2$, the midpoint in the disc $1/4$ (Bertrand's paradox) — chords from rods thrown at random on the plane matched choosing the midpoint on a radius — [Paper 630]

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— *From thinking to syncing.* —

Abstract

A chord is drawn "at random" in a circle of radius 1, and the probability that it is longer than $\sqrt{3}$, the side of the inscribed equilateral triangle, is sought.Three ways of placing "at random" are counted by simulation and formula, and a rod-throwing experiment on the plane is compared with them.No new theorem or law is claimed. First, the answer splits three ways——choosing both ends uniformly on the circle gives $1/3$; choosing a radius and placing the midpoint uniformly on it gives $1/2$; placing the midpoint uniformly in the disc gives $1/4$. With 2000000 random chords each, the results were 0.333147, 0.499918 and 0.249906 (Section 2). **Second, it is not only one threshold that splits——the mean chord length also splits, as $4/\pi = 1.273240$, $\pi/2 = 1.570796$ and $4/3$ by formula and 1.272753, 1.570556 and 1.333237 by simulation. The three placements differ in the whole distribution of chord lengths** (Section 3). **Third, and this is the core.The three answers differ in which quantity fixing the chord is taken as uniform——the positions of the ends, the distance from the centre to the chord, or the position of the midpoint. The chord is long when the distance from the centre is less than $1/2$, and the three placements assign this same event different probabilities. The words "a random chord" alone do not decide which is uniform** (Sections 2 and 5). **Fourth, the rod-throwing experiment matches choosing on a radius——taking a uniform point in a large square and drawing a line through it in a uniform direction, the 224442 chords from lines hitting the circle gave a share longer than $\sqrt{3}$ of 0.499688, a share longer than the radius 1 of 0.867102 (the formula for choosing on a radius is $\sqrt{3}/2 = 0.866025$), and a mean length of 1.571687 (likewise $\pi/2$); all three were closest, among the three formulas, to choosing on a radius. Fixing how to throw fixes the answer** (Section 4). **Keywords**: Bertrand paradox, geometric probability, random chord, choice of uniform measure, Jaynes.

Scope of this paper (scope note): **No new theorem or law is claimed——the three answers (Bertrand, 1889) and that lines thrown uniformly on the plane give $1/2$ (Jaynes's paper of 1973 and others) are known.Which answer is "correct" is not claimed——what is shown is only that one rod-throwing experiment matches choosing on a radius.The rod experiment is an approximation——lines through uniform points in a square of side 40 approximate lines uniform on the whole plane.Relation to earlier papers**: **Paper 607 showed that three different weights on the same tree of tic-tac-toe move sequences split the first player's winning share three ways——here three different uniforms on the chords of the same circle split the probability of a long chord three ways.Paper 611 showed that fixing how the host opens a door fixes the chance of winning by switching——here fixing how the rod is thrown fixes the answer.Paper 164, in its roll call of the addresses of probability, noted that Kolmogorov's axioms fix no value of probability——all three answers for "a random chord" satisfy the axioms.**What is added is **comparing the three placements by both a threshold probability and the mean length**, **matching the rod experiment against the three formulas in three quantities**, and **placing the separator on what is taken as uniform**.

1. Notation and the Question

Setting: **in a circle of radius 1, a chord at distance d from the centre has length $2\sqrt{1-d^2}$.The chord is longer than $\sqrt{3}$, the side of the inscribed equilateral triangle, when $d < 1/2$.**

Question: **what is the probability that a chord drawn "at random" is longer than $\sqrt{3}$?**

2. Three Placements

Computation 2.1: 2000000 **random chords were made by each of three placements.**

placement	uniform on	formula	simulation
ends	two points on the circle	$1/3$	0.333147
radius	midpoint on a radius (distance d)	$1/2$	0.499918
midpoint	midpoint in the disc	$1/4$	0.249906

Observation 2.2

in all three, a long chord is the same event, $d < 1/2$.With the ends placement, fixing one end, the chord is long when the other end falls in the opposite arc of $1/3$ of the circle. With the radius placement, d is uniform from 0 to 1, giving $1/2$. With the midpoint placement, the midpoint must fall in the circle of radius $1/2$, with probability equal to the area ratio $1/4$——different distributions of d assign the same event different probabilities.

3. Mean Lengths

Computation 3.1: **with the same random chords, the mean chord length and the share of chords longer than the radius 1 were found.**

placement	mean length (formula)	mean length (simulation)	share longer than radius (formula, simulation)
ends	$4/\pi = 1.273240$	1.272753	$2/3 = 0.666667$, 0.666401
radius	$\pi/2 = 1.570796$	1.570556	$\sqrt{3}/2 = 0.866025$, 0.865699
midpoint	$4/3 = 1.333333$	1.333237	$3/4$, 0.749811

Observation 3.2

the three placements differ not only in one threshold probability but in the whole distribution of chord lengths.The radius placement gives more long chords, the ends placement more short ones——changing the threshold from $\sqrt{3}$ to 1 leaves the three answers still apart.

4. Throwing Rods

Computation 4.1: **a uniform point was taken in a square of side 40 and a line drawn through it in a uniform direction. The 224442 chords from lines hitting the circle were compared with the three formulas in three quantities.**

quantity	rods	difference from ends	from radius	from midpoint
share longer than $\sqrt{3}$	**0.499688**	—	—	—
share longer than radius	0.867102	0.200435	0.001076	0.117102
mean length	1.571687	0.298447	0.000891	0.238354

The share longer than $\sqrt{3}$, 0.499688, is closest to $1/2$ among the three formulas $1/3$, $1/2$ and $1/4$. The other two quantities also differ least from the formulas of the radius placement.

> **Observation 4.2**
>
> **a line is fixed by two things, its direction and its distance from the centre.For a fixed direction, lines thrown uniformly on the plane have a uniform distance——which is the same as placing the midpoint uniformly on a radius. Fixing a procedure, how to throw, fixes what is uniform, and so fixes the answer.**

5. The Separator

> **Observation 5.1**
>
> **this is the core: the separator is what is taken as uniform.Whether the positions of the ends, the distance from the centre or the position of the midpoint is uniform splits the answer into $1/3$, $1/2$ and $1/4$——the words “a random chord” fix the set of chords but not the uniform on it.**

> **Observation 5.2**
>
> **beside Paper 607, Paper 611 and Paper 164.In Paper 607, three weights on the same move tree split the first player’s winning share three ways; in Paper 611, fixing how the host opens fixed the probability; Paper 164 noted that Kolmogorov’s axioms fix no value of probability——here all three answers satisfy the axioms, and only fixing the procedure of throwing the rod makes it one.**

Observation 5.3

what is not said.Which answer is “correct” is not claimed, and the argument that lines uniform on the plane are the one natural placement (Jaynes) is not judged——what is shown is the numbers for the three placements and which of them one experiment matches.

6. Roll Call and Closing

1. $1/3$ **by the ends** (Computation 2.1): **simulation** 0.333147**.**
2. $1/2$ **by the radius** (Computation 2.1): **simulation** 0.499918**.**
3. $1/4$ **by the midpoint** (Computation 2.1): **simulation** 0.249906**.**
4. **the same event** $d < 1/2$ (Observation 2.2): **different distributions of** d**.**
5. **three mean lengths** (Computation 3.1): $4/\pi$**,** $\pi/2$ **and** $4/3$**.**
6. **apart for another threshold too** (Observation 3.2): **the whole distribution differs.**
7. **rods give** 0.499688 (Computation 4.1): **closest to** $1/2$**.**
8. **all three quantities closest to the radius formulas** (Computation 4.1): **differences** 0.001076 **and** 0.000891**.**
9. **what is taken as uniform** (Observation 5.1): **the core.**
10. **fixed only by a procedure** (Observation 5.2): **Paper 607, Paper 611 and Paper 164.**

Closing: **For a chord drawn at random in a circle of radius** 1**, the probability of being longer than** $\sqrt{3}$ **was** $1/3$ **choosing both ends on the circle,** $1/2$ **choosing the midpoint on a radius, and** $1/4$ **choosing the midpoint in the disc.The three answers differ in which quantity fixing the chord is taken as uniform——the three placements assign the same event of a long chord different probabilities, and their mean lengths differ too.The rod-throwing experiment on the plane matched choosing on a radius in all three quantities——fixing the procedure fixes the answer.The separator is what is taken as uniform.Placed among the earlier papers——like the three weights of Paper 607 and the host’s rule of Paper 611, words alone do not fix a probability.To be honest——which answer is “correct” is not entered, and the rod experiment is an approximation by a large square.**

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