Adding a Road Slows Everyone Down Only When the Number of Cars Lies Between 3000 and 9000

— with 4000 cars taking 65 minutes on two routes, adding a 0-minute shortcut sends everyone through it for 80 minutes — it is slower only for more than 3000 and fewer than 9000 cars, the ratio of the settled state to the least total reaching 4/3 at 4500 — [Paper 631]

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— From thinking to syncing. —

Abstract

There are two routes from start to finish. One is a road taking the number of cars /100 minutes followed by a 45-minute road; the other is a 45-minute road followed by a road taking the number of cars /100 minutes. A 0-minute shortcut joining the middles of the two routes is added.Where each driver picks their own shortest route and no one gains by switching alone (the settled state) is counted as the number of cars changes (Braess's paradox).No new theorem or law is claimed. First, with 4000 cars, adding the shortcut turns 65 minutes into 80——before, the cars split in half and take 65 minutes. After, the settled state, found both by closed form and by a numerical method that does not assume symmetry (the Frank--Wolfe method), has everyone on the shortcut, taking 80 minutes. The other routes take 85, so no one switches (Section 2). **Second, an assignment minimising the total gains from the shortcut ——sending only 500 of the 4000 cars through the shortcut gives an average of 64.687500 minutes, shorter than the 65 before. With someone assigning routes, adding a road never makes things worse** (Section 2). **Third, and this is the core.Whether it gets slower is decided by the number of cars——checking every number from 1 to 12000 cars, the shortcut slows things for 3001 to 8999 cars, speeds them for 2999 or fewer (50 minutes become 20 with 1000 cars), and goes unused from 9000 cars** (Section 3). **Fourth, the ratio of the settled state to the least total reaches 4/3 at 4500 cars——the ratio is largest at 4500 cars, 90 minutes against 67.5, which is 1.333333. For roads whose times are linear in the number of cars this ratio is known never to exceed 4/3, and this network reaches that bound exactly** (Section 4). **Keywords**: Braess's paradox, traffic network, Wardrop equilibrium, price of anarchy, selfish routing.

Scope of this paper (scope note): **No new theorem or law is claimed——that adding a road can slow the settled state (Braess, 1968), the condition for the settled state (Wardrop, 1952), and that with linear times the ratio of the settled state to the least total never exceeds $4/3$ (Roughgarden and Tardos, 2002) are all known.The bound $4/3$ is not proved ——it is only counted that this network reaches it.Real roads and policies are not discussed——the network of 45 minutes and the number of cars $/100$ minutes is a common textbook example, not the numbers of Braess's original paper.Relation to earlier papers**: **Paper 437 showed with a two-player payoff table that each choosing the best lands the total at its lowest, placing the separator on "one at a time or together"——here adding one more road slows everyone, and only when the number of cars lies in a certain range.Paper 462 set aside in its scope note the design of roads, such as the number of lanes and merges——this paper takes up that seat, treating the addition of one road.**What is added is **finding the settled state by closed form and by a numerical method not assuming symmetry**, **counting the range of cars for which it gets slower, one car at a time**, and **placing the separator on the number of cars**.

1. Notation and the Question

The network: **the road from start to point A takes $x/100$ minutes, where x is the number of cars using it. From A to finish takes 45 minutes. From start to point B takes 45 minutes, and from B to finish $x/100$ minutes.The shortcut added is a 0-minute road from A to B. There are then three routes: start--A--finish, start--B--finish, and start--A--B--finish.**

The settled state: **every route in use takes the same time, and every unused route takes at least as long. No one gains by switching alone (Wardrop's condition).**

Question: **does adding the shortcut make the settled state faster or slower, and what decides which?**

2. With 4000 Cars

Computation 2.1: **before the shortcut, the cars split 2000 and 2000, and both routes take $20 + 45 = 65$ minutes.**

Computation 2.2: **the settled state after adding the shortcut was found by closed form and by a numerical method starting with everyone on one route (the Frank--Wolfe method, not assuming symmetry).Both put all 4000 cars on start--A--B--finish, taking $40 + 0 + 40 = 80$ minutes. The other two routes take $40 + 45 = 85$ minutes.**

> **Observation 2.3**
>
> **to each driver the shortcut always looks better.A road taking the number of cars /100 minutes takes 40 minutes even with everyone on it, shorter than the 45-minute road. So everyone chooses both such roads and joins them by the shortcut——and as a result all go from 65 to 80 minutes, 15 minutes slower.**

Computation 2.4: **the assignment minimising the total of everyone's time was also found by the same two routes.It is 1750, 1750 and 500 on the shortcut, with an average of 64.687500 minutes.**

> **Observation 2.5**
>
> **with someone assigning routes, adding a road never makes things worse.An assignment not using the shortcut can always be chosen, so the least total cannot rise when a road is added——what got slower is the settled state, where each chooses their own shortest.**

3. Changing the Number of Cars

Computation 3.1: **for different numbers of cars, the time before the shortcut, the settled state after it, and the average of the least total were set side by side (for the nine representative numbers, the settled state was found by both routes and agreed within 0.001 minutes).**

cars	before	settled after	least-total average	ratio
1000	50	20	20	1.000000
2000	55	40	40	1.000000
3000	60	60	56.25	1.066667
4000	65	80	64.6875	1.236715
4500	67.5	90	67.5	1.333333
6000	75	90	75	1.200000
8000	85	90	85	1.058824
9000	90	90	90	1.000000
10000	95	95	95	1.000000

Checking every number from 1 to 12000 cars, the shortcut slows things for 3001 to 8999 cars, 5999 cases, and speeds them for 1 to 2999 cars. The shortcut goes unused from 9000 cars.

> **Observation 3.2**
>
> this is the core: whether adding a road slows things is decided by the number of cars.With N cars, the time before is $N/200 + 45$ minutes, and with everyone on the shortcut it is $N/50$. Below 3000 cars the shortcut's $N/50$ is shorter, and adding it speeds things. Above 4500 cars some drivers avoid the shortcut, and the settled state stays at 90 minutes——at 9000 cars, where the $N/200 + 45$ minutes before catch up with 90, the shortcut goes unused and the difference vanishes.

4. The Largest Ratio

Computation 4.1: the settled time divided by the least-total average is largest at 4500 cars, **1.333333** (90 minutes against 67.5).

Observation 4.2

for roads whose times are linear in the number of cars, this ratio is known never to exceed $4/3$.This network reaches that bound exactly at 4500 cars——for networks of this form, this is as far as individual choices can drift from the least total.

5. The Separator

Observation 5.1

the separator is the number of cars.Adding the same shortcut to the same network speeds things for 2999 cars or fewer, slows them for 3001 to 8999, and changes nothing from 9000——"adding a road slows things" is a property not of the network alone but of the network together with the number of cars.

Observation 5.2

beside Paper 437 and Paper 462.In Paper 437, two players each choosing the best landed the total at its lowest, and the separator was "one at a time or together". Here too the least total (together) gains from the shortcut and the settled state (one at a time) loses——on top of that, it was counted that the loss happens only when the number of cars lies in a certain range. Of the design of roads that Paper 462 set aside in its scope note, only the addition of one road is treated here.

Observation 5.3

what is not said.Whether a real shortcut should be closed is not discussed, and the bound $4/3$ is not proved ——what is shown is the settled state and least total for one network, and the range of cars for which it gets slower.

6. Roll Call and Closing

1. **65 minutes become 80 with 4000 cars** (Computation 2.2): **two routes agree.**
2. **the other routes take 85** (Computation 2.2): **no one switches.**
3. **the shortcut always looks better** (Observation 2.3): **40 minutes even with everyone.**

4. **64.687500 minutes for the least total** (Computation 2.4): **only 500 on the shortcut.**
5. **an assigner never loses** (Observation 2.5): **the settled state is what got slower.**
6. **slower for 3001 to 8999 cars** (Computation 3.1): **counted one car at a time.**
7. **50 minutes become 20 with 1000 cars** (Computation 3.1): **faster when few.**
8. **decided by the number of cars** (Observation 3.2): **the core.**
9. **ratio 1.333333 at 4500 cars** (Computation 4.1): **reaches the bound 4/3.**
10. **one at a time loses, together gains** (Observation 5.2): **Paper 437 and Paper 462.**

Closing: **Adding a 0-minute shortcut to two routes joining a 45-minute road and a road taking the number of cars /100 minutes turned the settled state for 4000 cars, where each picks their own shortest, from 65 minutes into 80 .Assigning routes to minimise the total shrinks it to 64.687500 minutes with the shortcut, so what got slower is the side of individual choice.Whether it gets slower is decided by the number of cars——slower for 3001 to 8999 cars, faster for 2999 or fewer, and the shortcut unused from 9000.The ratio of the settled state to the least total reaches 4/3 at 4500 cars.The separator is the number of cars.Placed among the earlier papers——to "each one's best is everyone's worst" of Paper 437, the direction of adding a road and the range of cars are added.To be honest——neither real roads nor the proof of the bound 4/3 are entered.**

References

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