Mixing Two Losing Games Wins, but Fixing the Odds Regardless of Capital Makes the Mix Lose Too

— Game A wins with probability 0.495; game B wins with 0.095 when the capital is divisible by 3 and 0.745 otherwise — fixing B's odds at 0.495652 regardless of capital makes it lose by the same -0.008695, and the mix loses too, by -0.009348 — [Paper 632]

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— From thinking to syncing. —

Abstract

Two games paying $+1$ for a win and -1 for a loss are set up (Parrondo's paradox). A wins with probability $1/2-\varepsilon$; B wins with $1/10-\varepsilon$ if the capital is divisible by 3 and $3/4-\varepsilon$ otherwise, with $\varepsilon=0.005$.The expected gain per round is found exactly from the motion of the capital's remainder on division by 3, and checked by simulation.No new theorem or law is claimed. First, each game alone loses——the expected gain per round is -0.010000 for A and -0.008695 for B. Playing B alone, the capital is divisible by 3 for a share 0.383612 of the time, so much of the play is on the bad side with odds 0.095 (Section 2). **Second, mixing half and half at random wins——choosing A or B with equal chance each round gives an expected gain of $+0.015704$. Repeating AABB also wins, with $+0.014651$, but alternating AB loses, with -0.006738. Simulations of 2000000 rounds each agreed in sign for all four of A, B, random mixing and AABB** (Section 3). **Third, and this is the core.Mixing changes the distribution of the capital's remainder——with A mixed in, the share of time the capital is divisible by 3 falls from 0.383612 to 0.345070. Playing B under that distribution gives an expected gain of $+0.041408$ per round, and averaging it half and half with A's -0.010000 gives $+0.015704$** (Section 4). **Fourth, fixing the odds regardless of capital makes the mix lose ——B′, with B's odds fixed at B's effective odds 0.495652 regardless of capital, loses by the same -0.008695 as B. Mixing A and B′ half and half gives -0.009348, still a loss** (Section 5). **Keywords**: Parrondo's paradox, losing games, mixing games, Markov chain, capital-dependent game.

Scope of this paper (scope note): **No new theorem or law is claimed——that two losing games can be mixed into a winning one, and the set of odds used here (Harmer and Abbott, 1999), are known.Physical Brownian ratchets are not treated.No investment or gambling advice is given.It is not generalised to "one wins only when the odds depend on capital"——only one opposite side, B′ with odds independent of capital, is checked.Relation to earlier papers**: **Paper 298 showed that in a game with fixed odds per round, even a doubling strategy keeps the expectation exactly 0——here the odds of one game change with the capital's remainder, so the way of combining moves the expected gain per round.**What is added is **finding the expected gains exactly from the stationary distribution of the remainder and matching them with simulation**, **splitting out the expected gain of B played under the mixed distribution of the remainder**, and **placing the separator on whether the odds depend on the capital's remainder, checking the opposite side with B′**.

1. Notation and the Question

The games: **a win adds $+1$ to the capital, a loss -1. A wins with probability $p_A = 1/2 - \varepsilon$. B wins with $p_1 = 1/10 - \varepsilon$ if the capital is divisible by 3, and $p_2 = 3/4 - \varepsilon$ otherwise. Here $\varepsilon = 0.005$.**

Expected gain per round: **the remainder s of the capital on division by 3 runs through three states $0, 1, 2$. With π_s the long-run share of time at remainder s (the stationary distribution) and p_s the odds of the game played there, the expected gain per round is $\sum_s \pi_s(2p_s - 1)$.**

Question: **why can two losing games be mixed into a winning one, and what change stops it?**

2. Each Alone Loses

Computation 2.1: **A's expected gain per round is $2p_A - 1 = -0.010000$.B's expected gain is -0.810000 when the capital is divisible by 3 and $+0.490000$ otherwise; playing B alone, the capital is divisible by 3 for a share 0.383612, giving -0.008695 in all.**

Observation 2.2

B tends to fall onto its bad side.Where the capital is not divisible by 3 it climbs with odds 0.745, but climbing soon reaches a multiple of 3, where the bad coin with odds 0.095 pulls it back——remainder 0 is one of three states, yet its share is 0.383612, more than $1/3$.With $\varepsilon = 0$, B alone has an expected gain of 4×10^{-16}, which is 0 within rounding, and B is exactly fair.

3. Mixing

Computation 3.1: **for each way of mixing, the expected gain per round was found exactly from the stationary distribution and checked by 2000000 simulated rounds.**

play	stationary distribution	simulation
A alone	−0.010000	−0.010094
B alone	−0.008695	−0.008480
random half and half	**+0.015704**	+0.016104
repeating AABB	+0.014651	+0.016028
repeating AB	−0.006738	—

The differences between simulation and stationary distribution were 0.000094 for A, 0.000215 for B, 0.000400 for random mixing and 0.001377 for AABB, and all four agreed in sign.

> **Observation 3.2**
>
> **mixing does not always win.AABB wins, but alternating AB loses with −0.006738——how the games are mixed changes the motion of the remainder, and so the answer.**

4. The Distribution of the Remainder Changes

Computation 4.1: **mixing half and half at random, the share of time the capital is divisible by 3 becomes 0.345070 (0.383612 with B alone).Playing B under this distribution gives an expected gain of +0.041408 per round. Averaging it half and half with A's −0.010000 gives +0.015704, the expected gain of the mix.**

> **Observation 4.2**
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> **this is the core: A stirs the capital's remainder.A goes up or down nearly evenly regardless of capital, so it disturbs B's pull back to multiples of 3 and lowers the share of play on the bad coin——the same B, played at different remainders, turns from a losing game into one gaining +0.041408.**

5. B′, Independent of Capital

Computation 5.1: **B′ was made by fixing B's odds, regardless of capital, at B's effective odds when played alone, 0.495652.B′ alone has the same expected gain as B, -0.008695, and mixing A and B′ half and half gives -0.009348.**

> **Observation 5.2**
>
> **if the odds do not depend on capital, the mix loses.Mixing games with fixed odds gives a negative expected gain close to the average of the two, $(-0.010000 - 0.008695)/2$——even if the distribution of the remainder changes, the expected gain cannot move when the odds do not depend on the remainder.**

6. The Separator

> **Observation 6.1**
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> **the separator is whether the odds depend on the capital's remainder.If they do, the way of mixing changes the distribution of the remainder, and a winning combination can come out of two losers. If not, two losers stay losers however they are mixed——B and B′, with the same effective odds 0.495652, split into $+0.015704$ and -0.009348 when mixed with A.**

> **Observation 6.2**
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> **beside Paper 298.In Paper 298, a doubling strategy in a game with fixed odds per round still had expectation exactly 0——that the expectation cannot be moved by the way of betting belongs to the side where the odds do not depend on capital. Here the odds change with the capital's remainder, so the way of combining moves the expected gain.**

> **Observation 6.3**
>
> **what is not said.Physical Brownian ratchets are not treated, no investment or gambling advice is given, and it is not generalised to "one wins only when the odds depend on capital"——what is shown is the exact expected gains for one set of games and one opposite side, B′.**

7. Roll Call and Closing

1. **A -0.010000, B -0.008695** (Computation 2.1): **each alone loses.**
2. **B spends 0.383612 at remainder 0** (Observation 2.2): **pulled back to the bad side.**
3. **random mixing gives $+0.015704$** (Computation 3.1): **a win.**
4. **AABB wins, AB loses** (Observation 3.2): **the way of mixing matters.**
5. **simulation agrees in sign** (Computation 3.1): **2000000 rounds each.**
6. **mixing lowers remainder 0 to 0.345070** (Computation 4.1): **less bad side.**
7. **B under that distribution gives $+0.041408$** (Computation 4.1): **a loser turns winner.**
8. **A stirs the remainder** (Observation 4.2): **the core.**
9. **B′, independent of capital, still loses mixed, -0.009348** (Computation 5.1): **the opposite side.**
10. **fixed odds cannot be moved** (Observation 6.2): **Paper 298.**

Closing: **Game A with odds 0.495 and game B, whose odds change between 0.095 and 0.745 with whether the capital is divisible by 3, lose alone by -0.010000 and -0.008695, but mixed half and half at random they win by $+0.015704$ .Mixing lowers the share of time the capital is divisible by 3 from 0.383612 to 0.345070, and under that distribution B becomes a winning game at $+0.041408$.With B′, whose odds are fixed at 0.495652 regardless of capital, the mix still loses, by -0.009348——the separator is whether the odds depend on the capital's remainder.The way of mixing also matters: alternating AB loses.Placed among the earlier papers——"the way of betting cannot move the expectation" of Paper 298 belonged to the side where the odds do not depend on capital.To be honest——neither physical ratchets nor general conditions are entered; one set of games and one opposite side were counted.**

References

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