Paying Largest Coins First Always Uses the Fewest Coins in Yen, but One Too Many at 48 Pence in Old British Coins

— old British 1, 3, 6, 12 and 24 pence do not miss either, but adding the half crown of 30 pence pays 48 pence as $30+12+6$, three coins against the two of $24+24$ — whether a set misses shows at one point, where the added coin meets the earlier set — [Paper 633]

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September 2026

— *From thinking to syncing.* —

Abstract

Paying an amount by taking the largest coin as often as possible, then the next (the greedy method), is compared with the fewest coins (found for every amount by dynamic programming).Which sets of coins make it miss, and what happens when it does, are counted.No new theorem or law is claimed. First, yen never misses——with yen coins (1, 5, 10, 50, 100, 500) plus notes (1000, 2000, 5000, 10000), and with US coins (1, 5, 10, 25), every amount from 1 to 20000 was paid with the fewest coins. It is known that if a set misses, its first miss comes before the sum of its two largest coins (Kozen and Zaks); with the yen notes this sum is 15000, so yen never misses at any amount (Section 2). **Second, adding the half crown to old British coins misses at 48 pence——the pre-decimal British 1, 3, 6, 12 and 24 pence never miss, but with the half crown of 30 pence added, 48 pence is paid as $30+12+6$, three coins against the two of $24+24$. Up to 20000 it misses at 3993 amounts, each time by just one coin. The set 1, 3, 4 misses at 6, and the set 1, 5, 10, 20, 25 at 40** (Section 3). **Third, and this is the core.Adding one coin to a set that never misses can make it miss——adding 4 to 1, 3, adding 30 to the old British set, and adding 25 to 1, 5, 10, 20 all gave sets that miss. Adding the yen notes 2000, 5000 and 10000 in turn kept yen from missing** (Section 4). **Fourth, whether it misses shows at one point——let m be the added coin c divided by the earlier set's largest coin, rounded up, and pay only m times that largest coin by the greedy method. If it takes more than m coins the set misses; if not, it never misses (the one-point test of Magazine, Nemhauser and Trotter). For all six additions, this one-point test agreed with the check of every amount up to 20000** (Section 4). **Keywords**: change-making problem, greedy algorithm, minimum number of coins, coin system, pre-decimal British coinage.

Scope of this paper (scope note): **No new theorem or law is claimed——that some coin sets make the greedy method miss the fewest coins, that the first miss comes before the sum of the two largest coins (Kozen and Zaks, 1994), and the one-point test for adding a coin (Magazine, Nemhauser and Trotter, 1975) are all known.The one-point test is not proved——it is only matched against the check of every amount for six additions.The design of coin sets and the history of coins are not discussed.Relation to earlier papers**: **Paper 348 showed that on the same graph the greedy method is optimal for spanning trees and misses for tours——there the problem decided success; here, within the same change-making problem, the set of coins decides it.**What is added is **checking nine sets at every amount up to 20000**, **counting how the missing sets miss (how often, and by only one coin)**, and **matching the one-point test with the check of every amount, placing the separator on whether the added coin fits the earlier set**.

1. Notation and the Question

Setting: **a set of coins is fixed and an amount n is paid. The greedy method takes the largest coin as often as possible, then the next. Its count is $G(n)$ and the fewest coins $M(n)$. $M(n)$ was found for every amount from 1 to 20000 by dynamic programming, building up from small amounts "one coin plus the fewest for the rest".An amount with $G(n) > M(n)$ is called a miss.**

Question: **which sets miss, and what happens when they do?**

2. Sets That Never Miss

Computation 2.1: **$G(n)$ and $M(n)$ were compared at every amount from 1 to 20000.**

set of coins	misses	sum of two largest
yen coins $1, 5, 10, 50, 100, 500$	0	600
yen coins and notes (up to $1000, 2000, 5000, 10000$)	0	15000
US $1, 5, 10, 25$	0	35
$1, 3$	0	4
old British $1, 3, 6, 12, 24$	0	36
$1, 5, 10, 20$	0	30

Observation 2.2

if a set misses, its first miss comes before the sum of its two largest coins (Kozen and Zaks).Every sum here is below 20000, so not missing up to 20000 means not missing at any amount——yen coins and notes pay every amount with the fewest coins.

3. Sets That Miss

Computation 3.1: **by the same comparison, three sets missed.**

set of coins	first miss	greedy	fewest	misses up to 20000
$1, 3, 4$	6	$4 + 1 + 1$ (three)	$3 + 3$ (two)	4999
old British $1, 3, 6, 12, 24, 30$	48	$30 + 12 + 6$ (three)	$24 + 24$ (two)	3993
$1, 5, 10, 20, 25$	40	$25 + 10 + 5$ (three)	$20 + 20$ (two)	3995

Each first miss came before the sum of the two largest coins (7, 54 and 45).Wherever it missed, the greedy method used at most one coin more than the fewest, in every set. The old British set misses at every amount from 48 to 53 pence.

Observation 3.2

the greedy method spoils what follows by taking the largest coin.At 48 pence, taking 30 first leaves 18, for which there is no single coin, so it needs at least two, three in all. Without the 30, two 24s pay it exactly——the larger coin cuts in between a pair of smaller coins.

4. Adding One Coin

Computation 4.1: **one coin was added to a set that never misses, and the one-point test (with added coin c, earlier largest c_k and $m = \lceil c/c_k \rceil$, whether the greedy count for mc_k is at most m) was compared with the check of every amount up to 20000 (up to 2000 for the small sets).**

coin added to the earlier set	m	amount checked	greedy count	verdict (one point, every amount)
4 to $1, 3$	2	6	3	misses, misses
30 to old British $1, \ldots, 24$	2	48	3	misses, misses
25 to $1, 5, 10, 20$	2	40	3	misses, misses
2000 to yen (up to 1000)	2	2000	1	never, never
5000 to yen (up to 2000)	3	6000	2	never, never
10000 to yen (up to 5000)	2	10000	1	never, never

Observation 4.2

this is the core: whether a set misses is decided by whether the added coin fits the earlier set, and that shows at one point.The British 30 sits below 48, twice 24, and makes 48 start with 30 and take three coins. The yen 5000 pays 6000, three times 2000, as $5000 + 1000$, two coins, fewer than three——the same "largest first" is right or wrong depending on where the added coin sits.

5. The Separator

Observation 5.1

the separator is whether the added coin fits the earlier set.If it fits, the greedy method gives the fewest at every amount; if not, missing amounts appear. Whether it fits is decided at the single point mc_k——not the number of coins or the size of amounts, but the joint of the set is what matters.

Observation 5.2

beside Paper 348.In Paper 348, on the same graph, spanning trees were optimal by the greedy method and tours were missed; the kind of problem decided success——here, within the same change-making problem, the set of coins decides it. And even when it missed, it was by only one coin.

Observation 5.3

what is not said.The design of coin sets and the history of coins are not discussed, and the one-point test is not proved——what is shown is the check of every amount for nine sets, and the agreement of the one-point test with it for six additions.

6. Roll Call and Closing

1. **yen coins and notes never miss** (Computation 2.1): **at every amount up to 20000.**
2. **misses come before the sum of the two largest** (Observation 2.2): **so it holds at every amount.**
3. **old British with 30 misses at 48** (Computation 3.1): **three coins against two.**
4. **1, 3, 4 at 6, 1, 5, 10, 20, 25 at 40** (Computation 3.1): **the same kind of miss.**
5. **missing by only one coin** (Computation 3.1): **up to 20000.**
6. **the large coin cuts in** (Observation 3.2): **30 breaks $24 + 24$.**
7. **adding one coin can make a set miss** (Computation 4.1): **three examples.**
8. **adding notes to yen never misses** (Computation 4.1): **2000, 5000 and 10000.**
9. **one point agrees with every amount** (Observation 4.2): **the core.**
10. **the set, not the problem, decides** (Observation 5.2): **Paper 348.**

Closing: **Paying the largest coins first gave the fewest coins at every amount up to 20000 for yen coins and notes and for US coins, and since a set that misses does so before the sum of its two largest coins, these never miss at any amount.Adding the half crown of 30 to the old British 1, 3, 6, 12 and 24 pence pays 48 pence as $30 + 12 + 6$, three coins, more than the two of $24 + 24$.Whether a set misses is decided by whether the added coin fits the earlier set, and shows at the single point of m times the earlier largest coin——for six additions, the one-point test agreed with the check of every amount.The separator is whether the added coin fits the earlier set.Placed among the earlier papers——in Paper 348 the problem decided the success of the greedy method; here, within one problem, the set of coins decides it.To be honest——neither the proof of the one-point test nor the design of coin sets is entered.**

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