Splitting 5/121 by the Largest Unit Fraction First Reaches a 25-Digit Denominator, While Three Terms Need Only 363

— taking the largest unit fraction each time, $5/121$ takes five terms and the last denominator has 25 digits — three terms suffice with $1/33 + 1/121 + 1/363$, the largest denominator only 363 — [Paper 634]

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— *From thinking to syncing.* —

Abstract

A fraction is written as a sum of distinct unit fractions, fractions with numerator 1 (an Egyptian fraction).Taking each time the largest unit fraction not exceeding the rest (the greedy method) is compared with the shortest form found by exhaustive search.No new theorem or law is claimed. First, $5/121$ takes five greedy terms, reaching a 25-digit denominator——the denominators are 25, 757, 763309, 873960180913 and $1527612795642093418846225$, and the ratios of their numbers of digits to the previous one are 2.059527, 2.043246, 2.029934 and 2.025209, close to 2 (Section 2). **Second, with three terms the largest denominator need only be 363——exhaustive search found no two-term form of $5/121$ and 21 three-term forms. The one with the smallest largest denominator is $1/33 + 1/121 + 1/363$, and the largest denominators of the 21 forms range from 363 to 275275** (Section 3). **Third, and this is the core.The greedy method's miss shows in the size of the denominators more than in the number of terms——the next greedy denominator d' is always larger than $d(d-1)$ for the current denominator d. So once it misses, the digits nearly double at each step, and a difference of just two terms becomes the difference between 3 digits and 25** (Sections 2 and 4). **Fourth, for fractions with numerator 4 or 5, it misses now and then——counting reduced fractions with denominators up to 200, the greedy method used more terms than the fewest for 18 of the 98 with numerator 4 and 23 of the 156 with numerator 5. $4/49$ takes four greedy terms (the last denominator 9204734721) but is $1/14 + 1/98$ in two** (Section 5). **Keywords**: Egyptian fractions, unit fractions, greedy algorithm, Sylvester's sequence, growth of denominators.

> **Scope of this paper (scope note)**: **No new theorem or law is claimed——that every fraction can be written as a sum of unit fractions, that the greedy method takes no more terms than the numerator (since Fibonacci), and that its denominators grow fast (Sylvester, 1880) are all known.The Erdős--Straus conjecture (whether $4/n$ always has a three-term form) is not treated——it is only counted that fractions with numerator 4 and 5 and denominators up to 200 all had forms of at most three terms.Relation to earlier papers**: **Paper 633 counted that even for coin sets where the greedy method for change misses, it takes only one coin more than the fewest——here the same "largest first" makes the denominators explode when it misses.Paper 348 showed that on the same graph the greedy method is optimal for spanning trees and misses for tours——this is also on the missing side, and the size of the miss is measured.**What is added is **listing the 21 three-term forms of $5/121$ by exhaustive search**, **counting the ratios of the denominators' digits**, **counting how often fractions with numerator 4 and 5 miss**, and **placing the separator on whether the largest is taken without looking ahead**.

1. Notation and the Question

Setting: **a positive fraction r is written as a sum of distinct unit fractions $1/d_1 + 1/d_2 + \cdots$.The greedy method takes the largest unit fraction $1/d$ not exceeding r (d the smallest integer at least $1/r$) and repeats on the rest $r - 1/d$ .Exhaustive search, with denominators in increasing order, counts every way of writing it in k terms (with k terms the first denominator is at most k/r).**

Question: **when does the greedy method miss, and what happens when it does?**

2. The Greedy Method on 5/121

Computation 2.1: $5/121$ **was split by the greedy method, and the sum returned exactly $5/121$.**

term	denominator	digits
1	25	2
2	757	3
3	763309	6
4	873960180913	12
5	1527612795642093418846225	25

Comparing the common logarithm of each denominator with the previous gives 2.059527, 2.043246, 2.029934 and 2.025209: the digits nearly double.

> **Observation 2.2**
>
> the next greedy denominator is larger than $d(d-1)$ for the current denominator d.$1/d$ was taken because the rest was less than $1/(d-1)$, so what remains after taking it is less than $1/(d-1) - 1/d = 1/\{d(d-1)\}$. The next unit fraction does not exceed that, so its denominator is larger than $d(d-1)$——that the denominators nearly square, and the digits nearly double, is a property of the greedy method itself.

> **Observation 2.3**
>
> the number of terms does not exceed the numerator.The greedy method lowers the numerator of the rest each time, so numerator 5 gives at most five terms. $5/121$ uses all five. Counting denominators up to 200, fractions with numerator 4 took at most four terms and those with numerator 5 at most five.

3. Exhaustive Search

Computation 3.1: every way of writing $5/121$ in two or three terms was counted by exhaustive search.There is no two-term form and there are 21 three-term forms.The one with the smallest largest denominator is

$$\frac{5}{121} = \frac{1}{33} + \frac{1}{121} + \frac{1}{363}$$

and the largest denominators of the 21 forms range from 363 to 275275.

> **Observation 3.2**
>
> the first move differs.The greedy method first takes $1/25$, but the form with the smallest largest denominator starts with 33. The first move changes the fraction left over——the one move taken without looking ahead fixes every denominator after it.

4. The Size of the Miss

> **Observation 4.1**
>
> **this is the core: the greedy method's miss shows in the size of the denominators more than in the number of terms.For $5/121$ the terms are five against three, just two more, but the largest denominator has 25 digits against 363. As Observation 2.2 shows, once on the missing path the denominators nearly square at each step——the same "largest first" can miss by one coin, as in change, or miss explosively.**

5. Fractions with Numerator 4 and 5

Computation 5.1: **for reduced fractions $4/b$ and $5/b$ with denominators up to 200, the greedy number of terms was compared with the fewest found by exhaustive search (up to three terms).Every fraction had a form of at most three terms, so the fewest is exact.**

numerator	fractions	greedy uses more	examples (denominator, greedy, fewest)
4	98	18	**17** four, three; **49** four, two
5	156	23	**31** five, three; **121** five, three

Computation 5.2: $4/17$ **takes four greedy terms,** $1/5 + 1/29 + 1/1233 + 1/3039345$**, but three terms suffice:** $1/6 + 1/17 + 1/102$ **(there are 4 three-term forms).$4/49$ takes four greedy terms, with denominators 13, 213, 67841 and 9204734721, but is** $1/14 + 1/98$ **in two.**

> **Observation 5.3**
>
> **the misses come now and then.18 of 98 with numerator 4, and 23 of 156 with numerator 5——for most fractions the greedy method is shortest, but where it misses the denominators become larger by orders of magnitude.**

6. The Separator

Observation 6.1

the separator is whether the largest is taken without looking ahead.The greedy method takes the largest one step at a time, and exhaustive search chooses with the whole in view. When it misses, the difference shows not in the number of terms but in the digits of the denominators——for $4/49$, two terms against four, and a largest denominator of 98 against 10 digits.

Observation 6.2

beside Paper 633 and Paper 348.In Paper 633, even coin sets where the greedy method for change missed took only one coin more than the fewest. In Paper 348, the greedy method missed for tours——here the missing greedy method makes the denominators explode. Within "the greedy method misses" there are different faces, by the size of the miss.

Observation 6.3

what is not said.The Erdős--Straus conjecture is not treated, and forms longer than three terms were not searched exhaustively——what is shown is the greedy method and exhaustive search for $5/121$, $4/17$ and $4/49$, and how often fractions with numerator 4 and 5 miss.

7. Roll Call and Closing

1. **$5/121$ takes five greedy terms** (Computation 2.1): **the last denominator has 25 digits.**
2. **digit ratios close to 2** (Computation 2.1): **2.059527 to 2.025209.**
3. **the next denominator exceeds $d(d-1)$** (Observation 2.2): **a property of the greedy method.**
4. **terms do not exceed the numerator** (Observation 2.3): **$5/121$ uses all five.**
5. **no two-term form, 21 three-term forms** (Computation 3.1): **exhaustive search.**
6. **$1/33 + 1/121 + 1/363$** (Computation 3.1): **largest denominator 363.**
7. **the miss shows in the denominators** (Observation 4.1): **the core.**
8. **18 with numerator 4, 23 with numerator 5** (Computation 5.1): **misses now and then.**

9. **$4/49$: four terms against two** (Computation 5.2): **$1/14 + 1/98$.**

10. **change misses by a coin, fractions by digits** (Observation 6.2): **Paper 633 and Paper 348.**

Closing: **Splitting $5/121$ each time by the largest unit fraction not exceeding the rest takes five terms, with denominators $25, 757, 763309, 873960180913$ nearly doubling in digits, the last having 25 digits.Exhaustive search finds 21 three-term forms, and with $1/33 + 1/121 + 1/363$ the largest denominator need only be 363.The next greedy denominator exceeds $d(d-1)$ for the current d, so once it misses the denominators explode——the miss shows in the size of the denominators more than in the number of terms.For fractions with numerator 4 and 5, it misses now and then.The separator is whether the largest is taken without looking ahead.Placed among the earlier papers——in the change of Paper 633 a miss cost only one coin, but here the same greedy method makes the denominators explode.To be honest——neither the Erdős--Straus conjecture nor an exhaustive search of forms longer than three terms is entered.**

References

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