Cars Parked at Random Jam at 0.747598 of the Street, but Fill 0.864665 When Each Takes Two Cells

— the filled share is 0.747598 by Rényi's integral formula and 0.747457 by simulation — cutting the street into cells with each car taking k cells, $k = 1$ fills everything, $k = 2$ gives $1 - e^{-2} = 0.864665$, and $k = 1000$ falls to 0.747814 — [Paper 635]

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— From thinking to syncing. —

Abstract

Cars of length 1 are parked one at a time in the free places of a long street, each at a position chosen uniformly among those where it fits, and never moved. When no further car fits, the share of the street filled is sought (Rényi's parking problem).A street without breaks is compared with one cut into cells.No new theorem or law is claimed. First, a street without breaks jams at 0.747598——integrating Rényi's formula numerically gives 0.747598. Parking a street of length 100000 full, 20 times at random, gives an average of 0.747457, and the difference from the formula, 0.000140, is less than three times the standard error of the mean, 0.000110 (Section 2). **Second, cutting the street into cells changes the filled share——with each car taking k cells, the share from the recursion for the expected number of cars is 1.000000 for $k = 1$, 0.864665 for $k = 2$ (agreeing with the known $1 - e^{-2}$), 0.823653 for $k = 3$, 0.769577 for $k = 10$ and 0.747814 for $k = 1000$** (Section 3). **Third, and this is the core.The finer the street is cut relative to the car, the more gaps remain when it jams——as k grows the share falls steadily towards the 0.747598 of the street without breaks. Multiplying the difference from the formula by k gives 0.216543 for $k = 100$ and 0.216217 for $k = 1000$, nearly equal, so the difference shrinks roughly as $0.2165/k$** (Section 3). **Fourth, one cell per car fills everything——with $k = 1$, any empty cell can always take a car, so no gap remains. Once a car spans two cells or more, gaps shorter than a car remain unfilled however long one waits** (Sections 3 and 4). **Keywords**: Renyi parking constant, random sequential adsorption, Langmuir adsorption, lattice recurrence, Monte Carlo check.

Scope of this paper (scope note): **No new theorem or law is claimed——the share 0.747598 for a street without breaks and its integral formula (Rényi, 1958), the $1-e^{-2}$ for cars taking two cells (Flory, 1939), and the recursion for general k cells (collected in reviews of random sequential adsorption) are all known.Two-dimensional packing and cars that leave are not treated——only a one-dimensional street where parked cars never move is considered.Relation to earlier papers**: **Paper 284 showed that Langmuir adsorption, where each site takes only one molecule, is the same formula as the Fermi distribution——there the sites are independent, and raising the pressure brings the covered share close to 1. Here, once a car spans two cells or more, the sites are no longer independent, and the share never reaches 1 however long one waits.**What is added is **matching 0.747598 by three routes, the integral formula, the lattice recursion and simulation**, **changing k from 1 to 1000 and counting the approach as $0.2165/k$**, and **placing the separator on how finely the street is cut relative to the length of a car**.

1. Notation and the Question

A street without breaks: **cars of length 1 are parked one at a time on a street of length L. Each car goes to a position chosen uniformly among those not overlapping other cars, and is never moved. It ends when only gaps shorter than 1 remain. The filled share is considered in the limit of large L.**

A street cut into cells: **on a row of n cells, cars taking k consecutive cells are parked one at a time at positions chosen uniformly among those where they fit. With $E(n)$ the expected number of cars parked on a row of n cells, the first car splits the row into two, so**

$$E(n)=1+\frac{2}{n-k+1}\sum_{i=0}^{n-k}E(i)\qquad (n\ge k),\qquad E(n)=0\quad (n<k).$$

The filled share is $kE(n)/n$ for large n, found from the difference between n and $n/2$ to remove the effect of the ends.

Question: **when it jams, what share of the street is filled, and what decides it?**

2. A Street Without Breaks

Computation 2.1: **Rényi's integral formula**

$$c=\int_0^{\infty}\exp\left(-2\int_0^{t}\frac{1-e^{-u}}{u}\,du\right)dt$$

integrated numerically gives $\mathbf{0.747598}$ (estimated integration error 2.5×10^{-9}).

Computation 2.2: on a street of length 100000, cars were parked at uniform positions in each remaining gap until full. Repeating this 20 times, the filled share averaged 0.747457, with spread (standard deviation) 0.000491 and standard error of the mean 0.000110.The difference from the formula is 0.000140, less than three times the standard error, 0.000330.

> **Observation 2.3**
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> the street jams without being full.Gaps shorter than 1 remain between cars, and no further car fits in them ——everything beyond the filled 0.747598 is gaps too short for a car.

3. A Street Cut into Cells

Computation 3.1: with each car taking k cells, the filled share was found from the recursion.

k	1	2	3	5	10	100	1000	no breaks
filled share	1.000000	0.864665	0.823653	0.792276	0.769577	0.749763	0.747814	0.747598

$k = 2$ agreed with the known value $1 - e^{-2} = 0.864665$ (Flory) within 10^{-6}. As k grew, the share fell steadily.

Computation 3.2: multiplying the difference from the 0.747598 of the street without breaks by k gives 0.216543 for $k = 100$ and 0.216217 for $k = 1000$.The difference for $k = 1000$ is 0.000216.

> **Observation 3.3**
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> this is the core: the finer the street is cut relative to the car, the more gaps remain when it jams.With $k = 1$ any empty cell can always take a car, so everything fills. With $k = 2$ single-cell gaps remain, and as k grows the possible lengths of leftover gaps (from 1 to $k - 1$ cells) multiply, approaching the street without breaks ——since the difference times k settles near 0.2165, the difference shrinks roughly as $0.2165/k$.

4. The Separator

Observation 4.1

the separator is how finely the street is cut relative to the length of a car.One cell per car gives 1.000000, two cells 0.864665, and the limit of fine cutting 0.747598——the same “park at random and never move” jams differently depending on the cutting.

Observation 4.2

beside Paper 284.In the Langmuir adsorption of Paper 284, each site takes one molecule, the sites are independent, and raising the pressure brings the covered share close to 1. This is the $k = 1$ side——once a car spans two cells or more, whether it fits depends on whether the neighbouring sites are taken, and the sites are no longer independent. Then the share never reaches 1 however long one waits.

Observation 4.3

what is not said.Two-dimensional packing and cars that leave are not treated. That the difference shrinks as $0.2165/k$ is only counted at two values of k, with no formula given——what is shown is the agreement of three routes and the filled share as k changes.

5. Roll Call and Closing

1. **0.747598 by the integral formula** (Computation 2.1): **Rényi.**
2. **0.747457 by simulation** (Computation 2.2): **within three standard errors.**
3. **jammed but not full** (Observation 2.3): **gaps shorter than 1.**
4. **$k = 1$ fills everything** (Computation 3.1): **1.000000.**
5. **$k = 2$ gives 0.864665** (Computation 3.1): **agreeing with $1 - e^{-2}$.**
6. **falling steadily as k grows** (Computation 3.1): **towards 0.747598.**
7. **difference times k near 0.2165** (Computation 3.2): **0.216543 and 0.216217.**
8. **finer cutting leaves more gaps** (Observation 3.3): **the core.**
9. **the cutting decides the jam** (Observation 4.1): **the separator.**

10. **one molecule per site can reach 1** (Observation 4.2): **Paper 284.**

Closing: **Parking cars of length 1 at random on a long street and never moving them, it jams when 0.747598 of the street is filled.The integral formula, simulation, and the limit of streets cut into cells agreed on this value.Cutting the street into cells with each car taking k cells, $k = 1$ fills everything, $k = 2$ gives 0.864665, and $k = 1000$ falls to 0.747814——the difference from the formula shrinks roughly as $0.2165/k$.The separator is how finely the street is cut relative to the length of a car.Placed among the earlier papers——the one molecule per site of Paper 284 is the $k = 1$ side, where the covered share could approach 1.To be honest——neither two dimensions, nor cars that leave, nor a formula for $0.2165/k$ is entered.**

References

1. Evans, J. W. (1993). Random and cooperative sequential adsorption. *Reviews of Modern Physics*.
2. Flory, P. J. (1939). Intramolecular reaction between neighboring substituents of vinyl polymers. *Journal of the American Chemical Society*.
3. Yamagishi, Y. (2026). *The Langmuir Isotherm and the Fermi Distribution Are One and the Same Expression* [Paper 284]. Zenodo. 10.5281/zenodo.22171146

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