

The ABC Conjecture Is a Theorem for Polynomials and a Conjecture for Integers: Integers Lack the Derivative

— for polynomials **max deg** never exceeds the number of distinct roots minus 1 (the Mason--Stothers theorem), so the same inequality never fails — its proof uses that the Wronskian $ab' - a'b$ is divisible by the repeated part of the roots, and integers have no such derivative — [Paper 641]

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— *From thinking to syncing.* —

Abstract

For coprime positive integers $a + b = c$, write $\text{rad}(abc)$ for the product of the distinct primes dividing abc , and define the quality $q = \log c / \log \text{rad}(abc)$. The ABC conjecture says that for every $\varepsilon > 0$ only finitely many triples have $q > 1 + \varepsilon$. The same question can be posed for polynomials, where it is a theorem. This paper proves nothing; it counts both and sets them side by side. No new theorem or law is claimed. First, among integers there are triples with $q > 1$ —triples with $q > 1$ number 6 with $c < 10^2$, 31 with 10^3 , 120 with 10^4 , 418 with 10^5 and 1268 with 10^6 . Among the 151895 coprime triples with $c < 10^3$ there are 31, a share of 0.000204 (Section 2). Second, triples far above 1 are rare—of the 1268 triples with $c < 10^6$, 102 have $q > 1.2$, 11 have $q > 1.4$ and only 2 have $q > 1.5$; the highest is $1 + 4374 = 4375$ at 1.567887. The triple known for the highest quality, $2 + 3^{10} \times 109 = 23^5$, has **1.629912**. The family $1 + (3^{2^k} - 1) = 3^{2^k}$ keeps $q > 1$ for $k = 1$ to 6, but approaches 1, from 1.292030 at $k = 2$ to 1.056393 at $k = 6$ (Section 3). Third, and this is the core. For polynomials, the same inequality never fails—for coprime polynomials $a + b = c$, **max deg** does not exceed the number of distinct roots of abc minus 1 (the Mason--Stothers theorem). This inequality, the counterpart of $\log c \leq \log \text{rad}(abc)$ for integers, never failed in 600 triples built to have repeated roots, and the family $a = x^n$, $c = (x + 1)^n$ reached equality for $n = 2$ to 8 (Section 4). Fourth, the polynomial proof uses the derivative, and integers have no such derivative—the Wronskian $W = ab' - a'b$ is not 0 and is divisible $m - 1$ times by each root repeated with weight m . In all 600 triples, W was divisible by $abc/\text{rad}(abc)$. This step yields the polynomial inequality. Integers have nothing corresponding to a' , and the step does not carry over (Section 5). **Keywords:** abc conjecture, radical, Mason-Stothers theorem, polynomials, Wronskian.

Scope of this paper (scope note): No new theorem or law is claimed—the ABC conjecture (Masser and Oesterlé, around 1985), the theorem for polynomials (Stothers 1981, Mason 1984), the triple $2 + 3^{10} \times 109 = 23^5$ known for the highest quality, and counts of triples with $q > 1$ are all known. The ABC conjecture is not proved—only triples up to $c < 10^6$ are counted, and counting is not proof. The merits of Mochizuki’s claimed proof (2012) are not discussed—it is only noted that it has not been widely accepted in the mathematical community. It is not said that “with a derivative the integer case would be solved”—what is shown is only which step of the polynomial proof does not carry over to integers. Relation to earlier papers: Paper 236 measured the dimension of the solution set of a differential equation with the Wronskian—here the same Wronskian counts the repeated roots of polynomials. Paper 640 counted that the number of ways to write an even number as a sum of two primes carries factors set by division by small primes, and noted that counting is not proof—here too the meeting of an addition $a + b = c$ with the multiplicative structure $\text{rad}(abc)$ is counted. What is added is counting integer triples up to 10^6 while matching three ways of counting, checking the polynomial inequality and the divisibility of the Wronskian on random triples, and placing the separator on whether there is a derivative.

1. Notation and the Question

Integers: for coprime positive integers $a < b$ and $c = a + b$, let $\text{rad}(abc)$ be the product of the distinct primes dividing abc , and define the quality

$$q = \frac{\log c}{\log \text{rad}(abc)}.$$

$q > 1$ means $c > \text{rad}(abc)$. The ABC conjecture says that for every $\varepsilon > 0$ only finitely many triples have $q > 1 + \varepsilon$.

Polynomials: for coprime polynomials with complex coefficients $a + b = c$ (not all constant), let $n_0(abc)$ be the number of distinct roots of abc . The degree corresponds to the integer $\log c$, and $n_0(abc)$ to $\log \text{rad}(abc)$.

Question: why is the same question a theorem for polynomials and still a conjecture for integers? The difference is looked at in numbers.

2. Counting Integer Triples

Computation 2.1: triples were counted by exhaustive search for $c < 2000$, by a count that narrows candidates using $\text{rad}(a)\text{rad}(b)\text{rad}(c) < c$ up to $c < 10^5$, and by a faster version of the same narrowing up to $c < 10^6$; at each stage the two counts gave exactly the same triples.

range of c	$< 10^2$	$< 10^3$	$< 10^4$	$< 10^5$	$< 10^6$
triples with $q > 1$	6	31	120	418	1268

The 6 triples with $c < 100$ are $1 + 8 = 9$, $1 + 48 = 49$, $1 + 63 = 64$, $1 + 80 = 81$, $5 + 27 = 32$ and $32 + 49 = 81$. Of all 1501 coprime triples with $c < 100$, 6 have $q > 1$ (a share of 0.003997); with $c < 10^3$, 31 of 151895 (a share of 0.000204).

Observation 2.2

triples with $q > 1$ exist, but they are rare. In every one, one of a, b, c contains the same prime many times —for c to exceed $\text{rad}(abc)$, primes must repeat.

3. High-Quality Triples

Computation 3.1: the 1268 triples with $c < 10^6$ were ordered by quality.

triple	$\text{rad}(abc)$	q
$1 + 4374 = 4375$	210	1.567887
$343 + 59049 = 59392$	1218	1.547075
$121 + 255879 = 256000$	4290	1.488865
$37 + 32768 = 32805$	1110	1.482910
$1 + 2400 = 2401$	210	1.455673

Triples with q above 1.1, 1.2, 1.3, 1.4 and 1.5 number 379, 102, 29, 11 and 2.

Computation 3.2: the triple known for the highest quality, $2 + 3^{10} \times 109 = 23^5$ ($2 + 6436341 = 6436343$), was checked. It is coprime, with $\text{rad}(abc) = 15042$, $\log c = 15.677471$ and $\log \text{rad}(abc) = 9.618602$, giving $q = \mathbf{1.629912}$.

Computation 3.3: in $1 + (3^{2^k} - 1) = 3^{2^k}$, $3^{2^k} - 1$ is divisible by 2^{k+2} , so $\text{rad}(abc)$ falls below c . Since $3^{2^{k+1}} - 1 = (3^{2^k} - 1)(3^{2^k} + 1)$ and the second factor is even, the power of 2 grows by at least one with each step in k , and the family goes on without end. The divisibility was checked for $k = 1$ to 6.

k	1	2	3	4	5	6
q	1.226294	1.292030	1.235303	1.155623	1.095349	1.056393

Observation 3.4

there are infinitely many triples with $q > 1$, but the quality of this family approaches 1. What the ABC conjecture calls finite is the set with $q > 1 + \varepsilon$, not the set with $q > 1$ —the substance of the conjecture is not exceeding 1 but that exceeding 1 by a fixed margin cannot go on forever.

4. Polynomials

Computation 4.1: a and b were built from powers of small polynomials so as to have repeated roots, with $c = a + b$, and 600 coprime triples were made at random. In every triple $\max \deg \leq n_0(abc) - 1$ held; the smallest value of $(n_0(abc) - 1) - \max \deg$ was 1, and the largest of $\max \deg / (n_0(abc) - 1)$ was 0.875000.

Computation 4.2: with $a = x^n$, $c = (x + 1)^n$ and $b = c - a$, $n_0(abc) = n + 1$ for $n = 2$ to 8, reaching the equality $\max \deg = n_0(abc) - 1$.

Observation 4.3

this is the core: the inequality that fails for integers never fails for polynomials. For integers, $\log c \leq \log \text{rad}(abc)$ failed 1268 times with $c < 10^6$. The same inequality for polynomials, $\max \deg \leq n_0(abc) - 1$, never failed in 600 triples, reaching equality but never passing it—what remains a conjecture for integers is, for polynomials, a theorem that needs not even the margin ε .

5. The Key Step: the Wronskian

Observation 5.1

the polynomial proof uses the Wronskian

$$W = ab' - a'b = ac' - a'c$$

(the two forms agree because $c = a + b$). If a has a root x_0 with weight m , both a and a' are divisible by $(x - x_0)^{m-1}$, and so is W . The same holds for the roots of b from the first form and of c from the second. Being coprime, W is divisible by $abc/\text{rad}(abc)$. On the other hand, W has degree at most $\deg a + \deg b - 1$, so

$$\deg(abc) - n_0(abc) \leq \deg a + \deg b - 1,$$

which gives $\deg c \leq n_0(abc) - 1$ (and likewise for the other two).

Computation 5.2: in all 600 triples of Computation 4.1, W was not 0 and was divisible by $abc/\text{rad}(abc)$.

Observation 5.3

integers have nothing corresponding to a' . For polynomials, the derivative carries each repeated root into W “one fewer” times, where it can be set against the bound on the degree. For integers, there is at hand no operation that lowers the repetition of a prime by one and carries it into another number in a way that meshes with addition——this is where a step of the polynomial proof does not carry over to integers.

6. Fermat for Polynomials

Computation 6.1: for $n = 2$, $(1 - t^2)^2 + (2t)^2 = (1 + t^2)^2$ is a solution in coprime polynomials.

Observation 6.2

for $n \geq 3$ there is no solution in coprime polynomials. Applying Mason--Stothers to $x^n + y^n = z^n$, with D the largest degree of x, y, z , gives $nD \leq n_0(xyz) - 1 \leq 3D - 1$. With $D \geq 1$ this requires $n < 3$ ——Fermat for polynomials follows in one line. Fermat for integers took more than three hundred years, until Wiles's proof (1995).

7. The Separator

Observation 7.1

the separator is whether there is a derivative. Polynomials have one; the Wronskian counts repeated roots and turns the inequality into a theorem. Integers have no derivative in that role; the inequality fails 1268 times and survives as a conjecture with the margin ε —the same meeting of addition and repetition splits into theorem and conjecture according to whether there is a derivative.

Observation 7.2

beside Paper 236 and Paper 640. In Paper 236 the Wronskian measured the dimension of the solution set of a differential equation. In Paper 640 the number of ways to write an even number as a sum of two primes carried factors set by remainders on division—here the same Wronskian counts the repeated roots of polynomials and measures the meeting of the addition $a + b = c$ with the multiplicative structure $\text{rad}(abc)$. And as in Paper 640, counting on the integer side is not proof.

Observation 7.3

what is not said. The ABC conjecture is not proved, the merits of Mochizuki’s claimed proof are not discussed, and it is not said that “with a derivative the integer case would be solved”—what is shown is the count of integer triples, the checks of the polynomial inequality and the Wronskian, and which step of the polynomial proof does not carry over to integers.

8. Roll Call and Closing

1. 1268 triples with $q > 1$ for $c < 10^6$ (Computation 2.1): three ways of counting agree.
2. 31 of 151895 with $c < 10^3$ (Computation 2.1): rare.
3. 2 triples with $q > 1.5$ for $c < 10^6$ (Computation 3.1): the highest is 1.567887.
4. $2 + 3^{10} \times 109 = 23^5$ at **1.629912** (Computation 3.2): the known highest.
5. the family’s quality approaches 1 (Computation 3.3): 1.292030 to 1.056393.
6. the substance is the margin ε (Observation 3.4): not exceeding 1.
7. the polynomial inequality holds in 600 triples (Computation 4.1): with an equality family.

8. what fails for integers is a theorem for polynomials (Observation 4.3): the core.

9. W divisible by $abc/\text{rad}(abc)$ (Computation 5.2): the key step.

10. integers have no a' (Observation 5.3): Paper 236 and Paper 640.

Closing: For coprime $a + b = c$, triples whose quality $q = \log c / \log \text{rad}(abc)$ exceeds 1 number 1268 with $c < 10^6$, only 2 exceed 1.5, and the known highest, $2 + 3^{10} \times 109 = 23^5$, has 1.629912. For polynomials, the same inequality $\max \deg \leq n_0(abc) - 1$ never fails (the Mason--Stothers theorem)—its proof uses that the Wronskian $ab' - a'b$ is divisible by the repeated part of the roots, and integers have no such derivative. The separator is whether there is a derivative. Placed among the earlier papers—the Wronskian that measured the dimension of solutions in Paper 236 here counts repeated roots. To be honest—neither a proof of the ABC conjecture nor the merits of Mochizuki's claim is entered, and the count reaches only $c < 10^6$.

References

1. Granville, A., & Tucker, T. J. (2002). It's as easy as abc. *Notices of the American Mathematical Society*.
 2. Mason, R. C. (1984). *Diophantine Equations over Function Fields*. Cambridge University Press.
 3. Stothers, W. W. (1981). Polynomial identities and Hauptmoduln. *The Quarterly Journal of Mathematics*.
 4. Yamagishi, Y. (2026). *One and the Same “= 0” Returns Five Different Kinds of Collection* [Paper 236]. Zenodo. 10.5281/zenodo.22129043
 5. Yamagishi, Y. (2026). *The Number of Ways to Write an Even Number as a Sum of Two Primes Differs About Twofold by Whether It Is Divisible by 3* [Paper 640]. Zenodo.
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