In a Thin Layer Heated from Below, Surface Tension, Not Buoyancy, Drives the Flow, Trading Places at 25 mm

— in a 1 mm layer the surface-tension side starts at a temperature difference 646.98 times smaller, and at 50 mm buoyancy starts first — the separator is the thickness of the layer — [Paper 645]

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— *From thinking to syncing.* —

Abstract

When a liquid layer is heated from below, the liquid starts to move at some temperature difference, and seen from above it forms a pattern of cells (Bénard 1900). Two forces can drive it: warmed liquid becoming lighter and rising (buoyancy), and surface tension weakening on the warmed surface so that the surface is pulled toward the cooler side (the difference in surface tension).Which one moves the liquid first is counted for each thickness of the layer.No new theorem or law is claimed. First, the ratio of the two strengths moves with the square of the thickness——the strength of buoyancy (the Rayleigh number) goes with the cube of the thickness, and that of the surface-tension difference (the Marangoni number) with its first power. For water at 20 °C the two are equal at 8.7742 mm (Section 2). **Second, and this is the core.The side that starts first changes at about 25 mm——with the critical values from the literature (Rayleigh number 669.0, Marangoni number 79.607), the changeover thickness is 25.4358 mm, 2.898928 times the thickness of equal strength** (Section 3). **Third, in thin layers the surface-tension difference acts first by orders of magnitude——in a 1 mm layer, convection starts at a temperature difference of 47.3157 K by buoyancy alone and 0.0731331 K by the surface-tension difference alone, a ratio of 646.98. At 50 mm buoyancy started first** (Section 3). **Fourth, the separator is the thickness of the layer——with the same water heated the same way, the surface-tension difference moves the liquid if the layer is thinner than the changeover thickness, and buoyancy if it is thicker. The thin-layer cells that Bénard saw were explained not by buoyancy but by the surface-tension difference (Pearson 1958)** (Section 4). **Keywords**: Benard convection, Marangoni convection, Rayleigh number, Marangoni number, surface tension.

Scope of this paper (scope note): No new theorem or law is claimed——the onset of convection by buoyancy (Rayleigh 1916), the onset by the surface-tension difference (Pearson 1958), and their combined treatment (Nield 1964) are all known.The critical values come from the literature——the Rayleigh number 669.0 and Marangoni number 79.607 are values for one choice of boundaries, a rigid heat-conducting bottom and a flat heat-insulating free surface; the linear stability problem is not solved here. Other boundaries give other critical values.Surface contamination changes things——the temperature change of surface tension $d\sigma/dT$ is taken for clean water. Contaminants or surfactants on the surface weaken it and move the changeover thickness.Relation to earlier papers: Paper 452 showed that what lifts the liquid is not surface tension but its difference——here the same difference moves a heated layer.Paper 309 showed that the length dividing gravity from surface tension is nobody's size——here, in a heated layer, the thickness dividing buoyancy from the surface-tension difference is counted.What is added is deriving separately the thickness of equal strength and the thickness at which the side starting first changes, listing the onset temperature differences by thickness, and placing the separator on the thickness of the layer.

1. Notation and the Question

Setting: a liquid layer of thickness d is heated from below with a temperature difference ΔT across it. The strength of buoyancy is measured by the Rayleigh number and that of the surface-tension difference by the Marangoni number.

$$\mathrm{Ra} = \frac{g\beta\,\Delta T\,d^3}{\nu\kappa}, \qquad \mathrm{Ma} = \frac{|d\sigma/dT|\,\Delta T\,d}{\mu\kappa}$$

Here β is the thermal expansion coefficient, ν the kinematic viscosity, κ the thermal diffusivity, and $\mu = \rho\nu$ the viscosity.The premises are water at 20 °C: $\rho = 998.2$ kg/m^3, $\beta = 2.07 \times 10^{-4}$ /K, $\nu = 1.004 \times 10^{-6}$ m^2/s, $\kappa = 1.43 \times 10^{-7}$ m^2/s. Surface tension is taken as 72.74 mN/m at 20 °C and 71.18 mN/m at 30 °C, giving $d\sigma/dT = -1.5600 \times 10^{-4}$ N/(m·K).

Question: which moves a heated layer first, buoyancy or the surface-tension difference?

2. The Thickness of Equal Strength

Computation 2.1: the ratio $\mathrm{Ra}/\mathrm{Ma} = \rho g\beta d^2/|d\sigma/dT|$ does not depend on the temperature difference and moves with the square of the thickness. The thickness at which it is 1,

$$d^* = \sqrt{\frac{|d\sigma/dT|}{\rho g\beta}}$$

is 8.7742 mm for water at 20 °C.

> **Observation 2.2**
>
> **this length has the same form as the capillary length of Paper 309.The capillary length is the square root of surface tension σ divided by density and gravity. Here the temperature change of σ takes the place of σ, and the part of the density that changes with temperature, $\rho\beta$, takes the place of the density——both are lengths set by the balance of two forces, not by any size of the material.**

3. The Changeover of the Side That Starts First

Computation 3.1: **for either force, convection starts when the number exceeds its critical value. With the critical values from the literature, $\mathrm{Ra}_c = 669.0$ and $\mathrm{Ma}_c = 79.607$ (rigid heat-conducting bottom, flat heat-insulating free surface), the onset temperature difference was derived for each thickness.**

thickness	buoyancy alone	surface tension alone	ratio
0.5 mm	378.525 K	0.146266 K	2587.92
1 mm	47.3157 K	0.0731331 K	646.98
2 mm	5.91446 K	0.0365665 K	161.745
5 mm	0.378525 K	0.0146266 K	25.8792
10 mm	0.0473157 K	0.00731331 K	6.4698
25 mm	0.0030282 K	0.00292532 K	1.03517
50 mm	0.000378525 K	0.00146266 K	0.258792

The ratio is the buoyancy value divided by the surface-tension value; above 1, the surface-tension difference starts first.

Computation 3.2: **the thickness at which the two temperature differences are equal is 25.4358 mm by the closed form $d_c = d^{*}\sqrt{\mathrm{Ra}_c/\mathrm{Ma}_c}$ and 25.4358 mm by root-finding (agreeing within 10^{-9} m). It is 2.898928 times d^{*}.**

Observation 3.3

this is the core: the side that starts first changes at about 25 mm.The thickness of equal strength, 8.7742 mm, differs from the changeover, 25.4358 mm, because the two critical values differ: buoyancy must raise its number to 669.0 to move the liquid, the surface-tension difference only to 79.607——"which force is stronger" and "which moves the liquid first" are different questions.

4. The Separator

Observation 4.1

the separator is the thickness of the layer.With the same water heated the same way, at 1 mm the surface-tension difference starts first at a temperature difference 646.98 times smaller, and at 50 mm buoyancy starts first——below or above the changeover thickness of 25.4358 mm, what moves the liquid changes.

Observation 4.2

Bénard's cells.Bénard (1900) saw hexagonal cells in thin liquid layers. Rayleigh (1916) set out to explain them by buoyancy, but Pearson (1958) showed that in thin layers the surface-tension difference acts first ——in this paper's table too, at millimetre thickness the surface-tension side is first by orders of magnitude.

Observation 4.3

beside Paper 452.Paper 452 showed that what lifts liquid, as in the tears of wine, is not surface tension itself but its difference——here too a surface-tension difference made by temperature moves the liquid, and the thinner the layer, the more that side wins.

Observation 4.4

what is not said.The critical values are literature values for one choice of boundaries and are not solved here. Surface contamination weakens the temperature change of surface tension——what is shown is the changeover with thickness for clean water and one choice of boundaries.

5. Roll Call and Closing

1. **ratio moves with thickness squared** (Computation 2.1): **independent of temperature difference.**
2. **equal strength at 8.7742 mm** (Computation 2.1): **water at 20 °C.**
3. **same form as the capillary length** (Observation 2.2): **Paper 309.**
4. **ratio 646.98 at 1 mm** (Computation 3.1): **surface tension first.**
5. **ratio 0.258792 at 50 mm** (Computation 3.1): **buoyancy first.**
6. **changeover at 25.4358 mm** (Computation 3.2): **closed form and root-finding.**
7. **2.898928 times d^*** (Computation 3.2): **square root of the ratio of critical values.**
8. **stronger and first are different questions** (Observation 3.3): **the core.**
9. **thickness is the separator** (Observation 4.1): **thinner, surface tension; thicker, buoyancy.**
10. **Bénard's cells on the surface-tension side** (Observation 4.2): **Pearson.**

Closing: **Two forces move a liquid layer heated from below: buoyancy as warmed liquid rises, and the surface-tension difference as the warmed surface weakens.The ratio of their strengths moves with the square of the thickness, and for water at 20 °C they are equal at 8.7742 mm.Because the critical values differ, the side that starts first changes at 25.4358 mm, and in a 1 mm layer the surface-tension difference moves the liquid first at a temperature difference 646.98 times smaller——the thin-layer cells Bénard saw were on the surface-tension side.The separator is the thickness of the layer.Placed among the earlier papers——the surface-tension difference of Paper 452 here moves a heated layer.To be honest——the critical values come from the literature, and surface contamination would move the changeover thickness.**

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