The Spread of the Drake Equation Is Set Almost Entirely by the Factors Observation Cannot Yet Bound

— of the variance of the logarithm, the fraction on which life arises takes **0.665255** and the lifetime of communication **0.239492**, while the three factors observation can bound take **0.008770** together — the separator is whether a factor can be bounded by observation — [Paper 646]

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— From thinking to syncing. —

Abstract

The Drake equation writes the number N of civilizations now able to communicate in the galaxy as a product of seven factors: the rate of star formation, the fraction of stars with planets, the number of habitable planets, the fraction on which life arises, the fraction reaching intelligence, the fraction that communicates, and the lifetime of communication.Multiplication multiplies not only values but uncertainties. The width of each factor is set as a premise, and the source of the answer's spread is counted.No new theorem or law is claimed. Nothing is claimed about whether civilizations exist beyond Earth. First, the answer spans 22.778151 orders of magnitude——with the widths set here, the product with all seven factors at their low ends is 5×10^{-15} and at their high ends 3×10^{8}, a spread of 22.778151 orders (Section 2). **Second, and this is the core.The answer's spread is set almost entirely by the factors observation cannot yet bound——taking each factor uniform in the logarithm, the shares of the variance of $\log_{10} N$ are 0.665255 for the fraction on which life arises, 0.239492 for the lifetime of communication, 0.059873 for intelligence and 0.026610 for communication. The three that observation can bound (stars, planets, habitable planets) take 0.008770 together, and the four it cannot take 0.991230** (Section 3). **Third, "the share with N below 1" is an answer about the widths set——with 400000 random draws, the median of N was 0.001211 and the share with N below 1 was 0.775000. Changing only the lower end of the fraction on which life arises from 10^{-10} to 10^{-30} made that share 0.925480** (Section 4). **Fourth, the separator is whether a factor can be bounded by observation ——however precisely the three observable factors are measured, the answer's spread shrinks only by their share of 0.008770** (Section 5). **Keywords**: Drake equation, propagation of uncertainty, log-uniform distribution, extraterrestrial civilisations, variance decomposition.

Scope of this paper (scope note): **No new theorem or law is claimed——the Drake equation (1961) and the treatment that multiplies each factor's uncertainty in the logarithm to obtain a distribution for the answer (Sandberg, Drexler and Ord 2018) are both known.Nothing is claimed about whether civilizations exist beyond Earth——the widths are premises, not facts. What is counted is how the answer scatters within the widths set.The Fermi paradox is not resolved or discussed**.**The shape of the distribution is also a premise——each factor is taken uniform in the logarithm; other shapes move the values.Relation to earlier papers**: **Paper 292 showed that only the logarithm adds——here too, the uncertainty of a product adds as the sum of the variances of the logarithms.Paper 555 showed that under multiplication, fixing a stage that is not the bottleneck still helps——the same multiplication makes, for uncertainty, narrowing a narrow factor remove only its own share.**What is added is **checking the breakdown of the variance by closed form against random draws**, **showing that moving one factor's lower end moves the share in the answer**, and **placing the separator on whether a factor can be bounded by observation**.

1. Notation and the Question

Setting: **the Drake equation is**

$$N = R_* \, f_p \, n_e \, f_l \, f_i \, f_c \, L$$

where R_* is the number of stars formed in the galaxy per year, f_p the fraction with planets, n_e the number of habitable planets per planetary system, f_l the fraction on which life arises, f_i the fraction reaching intelligence, f_c the fraction that communicates, and L the lifetime of communication in years.

Widths (premises): R_* **1 to 3,** f_p **0.5 to 1, and** n_e **0.1 to 1 can be bounded by observations of stars and planets.** f_l 10^{-10} **to 1,** f_i 10^{-3} **to 1,** f_c 10^{-2} **to 1, and** L 10^2 **to** 10^8 **years cannot yet be bounded by observation.These widths are premises set by this paper; other widths move the answer.**

Question: **from which factors does the answer's spread come?**

2. The Spread of the Product

Computation 2.1: **with all seven factors at their low ends the product is** 5×10^{-15}**, and at their high ends** 3×10^{8}**, a spread of 22.778151 orders of magnitude.**

Observation 2.2

multiplication multiplies the widths too.Each factor's own width is at most 10 orders. With seven multiplied, the ends lie 22.778151 orders apart——saying "the answer is N" as one number hides this spread.

3. The Breakdown of the Variance

Computation 3.1: $\log_{10} N$ is the sum of the logarithms of the seven factors. Taking each factor uniform in the logarithm, the variance of each logarithm is one twelfth of the square of its logarithmic width, and the variance of the sum is the sum of the variances.

factor	variance of log	share
R_*	0.0190	0.001514
f_p	0.0076	0.000603
n_e	0.0833	0.006653
f_l	8.3333	0.665255
f_i	0.7500	0.059873
f_c	0.3333	0.026610
L	3.0000	0.239492

The three observable factors take 0.008770 together, and the four not yet observable take 0.991230.

Computation 3.2: building $\log_{10} N$ from 400000 random draws gave a mean of -2.908933 (closed form -2.911954) and a variance of 12.504821 (closed form 12.526522); the means differ by less than 0.02 and the ratio of variances lies between 0.99 and 1.01.

> **Observation 3.3**
>
> this is the core: the answer's spread is set almost entirely by the factors observation cannot yet bound.The fraction on which life arises alone takes 0.665255, and with the lifetime of communication the two take most of the rest. Observation of stars and planets has advanced greatly in the last half century, yet those three take 0.008770——the factors that are well measured and the factors that set the answer are different.

4. An Answer About the Widths Set

Computation 4.1: with 400000 random draws, the median of N was 0.001211 and the share with N below 1 was 0.775000.

Computation 4.2: **changing only the lower end of the fraction on which life arises from 10^{-10} to 10^{-30} made the median 1.271×10^{-13} and the share with N below 1 0.925480.**

> **Observation 4.3**
>
> **“the share with N below 1” is not an answer about the universe but about the widths set.Moving one factor’s lower end moved the share from 0.775000 to 0.925480. The widths of the factors not yet observable are set not by observation but by whoever sets them——which is why this paper says neither that civilizations beyond Earth exist nor that they do not.**

5. The Separator

> **Observation 5.1**
>
> **the separator is whether a factor can be bounded by observation.However precisely the three observable factors are measured, the variance of the logarithm of the answer falls only by their share of 0.008770. What would narrow the answer is a means of bounding the fraction on which life arises or the lifetime of communication——with the form of the equation unchanged, which factors can be reached decides how certain the answer is.**

> **Observation 5.2**
>
> **beside Paper 292 and Paper 555.Paper 292 showed that only the logarithm adds; here too, the uncertainty of the product added as the sum of the variances of the logarithms. Paper 555 showed that under multiplication, fixing a stage that is not the bottleneck still helps——the same multiplication separates each factor’s effect from the others. For values, that makes fixing any stage help; for uncertainty, it makes narrowing a narrow factor remove only its own share.**

> **Observation 5.3**
>
> **what is not said.Whether civilizations exist beyond Earth, the resolution of the Fermi paradox, and the true values of the factors are not discussed——what is shown is from which factors the answer’s spread comes, within the widths set.**

6. Roll Call and Closing

1. **product spread 22.778151 orders** (Computation 2.1): **multiplication multiplies widths.**
2. **f_i share 0.665255** (Computation 3.1): **the largest.**
3. **L share 0.239492** (Computation 3.1): **the second.**
4. **three observable factors 0.008770** (Computation 3.1): **small.**
5. **four not yet observable 0.991230** (Computation 3.1): **nearly all.**
6. **closed form and random draws agree** (Computation 3.2): **mean and variance.**
7. **well measured and answer-setting factors differ** (Observation 3.3): **the core.**
8. **share with N below 1 0.775000** (Computation 4.1): **a value for the widths set.**
9. **moving a lower end gives 0.925480** (Computation 4.2): **an answer about the widths.**
10. **whether observable is the separator** (Observation 5.1): **Paper 292 and Paper 555.**

Closing: **The Drake equation only multiplies seven factors, and multiplication multiplies not only values but uncertainties.With the widths set here, the ends of the answer lie 22.778151 orders apart, and 0.991230 of the variance of its logarithm comes from the four factors observation cannot yet bound.The three observable factors take 0.008770, and the share with N below 1 moved from 0.775000 to 0.925480 when one lower end was moved——that is an answer not about the universe but about the widths set.The separator is whether a factor can be bounded by observation.Placed among the earlier papers——the sum of logarithms of Paper 292 adds the uncertainties, and the multiplication of Paper 555 separates the effects factor by factor.To be honest——the widths and the shape of the distribution are premises, and whether civilizations exist beyond Earth is not entered.**

References

1. Sandberg, A., Drexler, E., & Ord, T. (2018). Dissolving the Fermi paradox. *arXiv*.
2. Shklovskii, I. S., & Sagan, C. (1966). *Intelligent Life in the Universe*. Holden-Day.
3. Yamagishi, Y. (2026). *Only the Logarithm Adds* [Paper 292]. Zenodo. 10.5281/zenodo.22171519
4. Yamagishi, Y. (2026). *Under Multiplication, Fixing a Stage That Is Not the Worst Still Helps* [Paper 555]. Zenodo. 10.5281/zenodo.23014208

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