# Neptune Was Found Not by a Number Series but by Gravity, and at Mercury It Was Gravity That Changed

— Bode's law fitted Mercury to Uranus and missed Neptune by **+29.0323** % — the separator is fit versus mechanism, and whether a residual can be absorbed by adding a body — [Paper 647]

Yuuki YAMAGISHI (山岸 勇輝)
Yamagishi Complexity Torus Lab, Japan
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— *From thinking to syncing.* —

**Abstract**

**There were two ways to predict planetary distances. One was to find a number series that fits the distances (Bode's law). The other was to compute orbits from the law of gravity and seek what explains the part that does not fit, the residual (Le Verrier).Where each way hit and where it missed is counted.No new theorem or law is claimed. First, the series also fitted planets found after it was written——$a = 0.4 + 0.3 \times 2^k$ astronomical units fitted the 8 bodies from Mercury to Uranus (Ceres included) with a largest miss of 4.9869 % (Mars). Both Uranus and Ceres were found after the series was written** (Section 2). **Second, the series missed Neptune——Neptune lies at 30.07 for the position 38.8, a miss of +29.0323 %, 5.821732 times the largest before it. Pluto, at the same position, fits within −1.7224 %, but as an agreement with no mechanism** (Section 2). **Third, and this is the core.Neptune was found not by the series but by the law of gravity and a residual, and the same move failed at Mercury——adding an unseen planet to Uranus's residual found Neptune. Adding an unseen planet (Vulcan) to the residual of Mercury's perihelion was tried, but none was found, and the law of gravity changed instead. The general-relativistic perihelion advance comes out from constants as 42.9807 arcseconds per century** (Section 3). **Fourth, the separator has two stages——fit or mechanism; and, within mechanism, whether the residual can be absorbed by adding a body or the law must change. At Uranus's distance the general-relativistic effect is 0.002385 arcseconds, so changing the law would not have removed Uranus's residual** (Section 4). **Keywords**: Titius-Bode law, discovery of Neptune, perihelion precession of Mercury, general relativity, Vulcan.

**Scope of this paper (scope note)**: **No new theorem or law is claimed——the Titius--Bode law (1766, 1772), the discovery of Neptune from Uranus's residual (1846), the search for Vulcan, and the explanation of Mercury's perihelion advance by general relativity (Einstein 1915) are all known history.Whether Bode's law has physical meaning is not discussed——this paper only counts the pattern of misses.Predicted positions are not copied——the numbers of Le Verrier's and Adams's predicted positions for Neptune are not written, as they were not checked.The semi-major axes are premises——rounded values in astronomical units are used.Relation to earlier papers**: **Paper 481 showed that a broken law can be blamed on any of the assumptions and that logic alone does not decide——here is a case where the same way of blaming (adding an unseen body) hit once and missed once.Paper 79 showed that inverse-square orbits close because of a hidden conserved quantity, and named Mercury's perihelion as an example of its non-conservation——here that perihelion advance is counted from constants.**What is added is **listing the misses of Bode's law through Neptune and Pluto**, **deriving the general-relativistic perihelion advance of Mercury and Uranus by the same formula**, and **placing the separator in two stages**.

## 1. Notation and the Question

**Setting**: **Bode's law lines up the semi-major axes of the planets (astronomical units) as**

$$a = 0.4 + 0.3 \times 2^k \qquad (k = 0, 1, 2, \ldots)$$

**with Mercury alone at 0.4. The premised semi-major axes are Mercury 0.387, Venus 0.723, Earth 1.000, Mars 1.524 , Ceres 2.767, Jupiter 5.203, Saturn 9.537, Uranus 19.19, Neptune 30.07 and Pluto 39.48.**

**Perihelion advance**: **general relativity advances the perihelion per orbit by**

$$\Delta\varphi = \frac{6\pi GM}{c^2 a(1 - e^2)}$$

**($GM$ the Sun's gravitational parameter, $e$ the eccentricity).**

**Question**: **where did fitting a series and computing from a law each hit and miss?**

## 2. The Misses of the Series

**Computation 2.1**: **the miss was taken as (Bode value minus actual) divided by actual.**

| body | $k$ | Bode | actual | miss |
|---|---|---|---|---|
| Mercury | — | 0.400 | 0.387 | +3.3592 % |
| Venus | 0 | 0.700 | 0.723 | −3.1812 % |
| Earth | 1 | 1.000 | 1.000 | 0.0000 % |
| Mars | 2 | 1.600 | 1.524 | +4.9869 % |
| Ceres | 3 | 2.800 | 2.767 | +1.1926 % |
| Jupiter | 4 | 5.200 | 5.203 | −0.0577 % |
| Saturn | 5 | 10.000 | 9.537 | +4.8548 % |
| Uranus | 6 | 19.600 | 19.190 | +2.1365 % |
| Neptune | 7 | 38.800 | 30.070 | +29.0323 % |
| Pluto | 7 | 38.800 | 39.480 | −1.7224 % |

**Among the 8 bodies from Mercury to Uranus the largest miss is 4.9869 % (Mars), and Neptune's miss is 5.821732 times that.**

> **Observation 2.2**
>
> **the series fitted bodies found after it was written, and then missed Neptune.Uranus entered the position 19.6 within 2.1365 % and Ceres the position 2.8 within 1.1926 %. But Neptune lies at 30.07 for the position 38.8, a miss 5.821732 times the largest of the others——Pluto enters the same position 38.8 within −1.7224 %, but the series has no mechanism to explain it. That hits are no evidence of a mechanism is shown by how the series misses.**

## 3. What Explains a Residual

**Observation 3.1**

**what found Neptune was not the series but the law of gravity.Uranus drifted little by little from the orbit computed from Newtonian gravity and the known planets. Le Verrier and Adams explained this residual by the pull of an unseen outer planet and computed its position. Galle found Neptune near it (1846)——the move of "adding an unseen body" to a residual hit.**

**Observation 3.2**

**the same move missed at Mercury.Mercury's perihelion also advanced slightly faster than computed with the pull of all known planets. Le Verrier, with the same move, placed an unseen planet (Vulcan) inside Mercury's orbit. It was searched for and not found——what removed this residual was not an added body but a change of the law of gravity itself (general relativity).**

**Computation 3.3: from the formula above, with the Sun's gravitational parameter and the speed of light, the perihelion advance of Mercury ($a = 0.387098$, $e = 0.205630$, period $87.9691$ days) is $42.9807$ arcseconds per century (agreeing with the known $43$ arcseconds, between $42.9$ and $43.1$). For Uranus ($a = 19.1913$, $e = 0.047168$, period $30688.5$ days) it is $0.002385$ arcseconds, less than $10^{-4}$ times Mercury's.**

**Observation 3.4**

**this is the core.Neptune was found not by fitting a series but from a law and a residual. And the same move of "adding a body to the residual" missed at Mercury, where the law changed instead——fitting and mechanism are different ways, and within mechanism there are residuals that an added body settles and residuals that require a change of law.**

## 4. The Separator

**Observation 4.1**

**the first separator is fit versus mechanism.A series has no reason for fitting, so when it misses it does not say what to fix. The law of gravity makes one ask where the unfitting part comes from——at Neptune's miss the series stopped, and the law pointed to the next body.**

**Observation 4.2**

**the second separator is whether the residual can be absorbed by adding a body.At Uranus's distance the general-relativistic effect is 0.002385 arcseconds, so changing the law would not remove Uranus's residual; a body was what had to be added. At Mercury no body was found, and a change of law with an effect of 42.9807 arcseconds removed the residual——with residuals of the same form, the distance from the Sun decided which kind of fix works.**

**Observation 4.3**

**beside Paper 481 and Paper 79.Paper 481 showed that a broken law can be blamed on any assumption and that logic alone does not decide. Here the same way of blaming hit at Uranus and missed at Mercury, and what decided was not logic but observation of whether a body lay where predicted. Paper 79 showed that inverse-square orbits close because of a hidden conserved quantity and named Mercury's perihelion as an example of its non-conservation——this paper counts that perihelion advance from constants as 42.9807 arcseconds.**

**Observation 4.4**

**what is not said.Whether Bode's law has physical meaning is not discussed. The numbers of Neptune's predicted position were not checked and are not written——what is shown is the pattern of the series' misses and the contrast between two ways of fixing a residual.**

## 5. Roll Call and Closing

1. **largest miss 4.9869 % over 8 bodies** (Computation 2.1): **Mars.**
2. **Uranus 2.1365 %, Ceres 1.1926 %** (Computation 2.1): **later finds also fit.**
3. **Neptune +29.0323 %** (Computation 2.1): **a miss.**
4. **5.821732 times the largest** (Computation 2.1): **a miss of another size.**
5. **Pluto −1.7224 %** (Observation 2.2): **agreement with no mechanism.**
6. **Neptune from law and residual** (Observation 3.1): **adding a body hits.**
7. **no Vulcan at Mercury** (Observation 3.2): **the same move misses.**
8. **Mercury 42.9807 arcseconds** (Computation 3.3): **a change of law.**
9. **Uranus 0.002385 arcseconds** (Computation 3.3): **changing the law would not remove it.**
10. **fit or mechanism, body or law** (Observations 4.1 and 4.2): **a two-stage separator.**

**Closing**: **There were two ways to predict planetary distances: fitting a series, and computing from a law and explaining the residual.Bode's series fitted the 8 bodies from Mercury to Uranus within 4.9869 % but missed Neptune by 29.0323 %.Neptune was found by a gravitational computation that added an unseen planet to Uranus's residual. The same move, searching for Vulcan, missed at Mercury, and the law of gravity changed instead; that perihelion advance comes out from constants as 42.9807 arcseconds, against only 0.002385 at Uranus——among residuals of the same form, some are settled by adding a body and some require changing the law.The separator is fit versus mechanism, and whether a residual can be absorbed by adding a body.Placed among the earlier papers——to Paper 481's "any assumption can be blamed", a case where the blame hit and a case where it missed are added.To be honest——the meaning of Bode's law is not discussed, and the numbers of Neptune's predicted position were not checked.**

## References


1. Baum, R., & Sheehan, W. (1997). *In Search of Planet Vulcan: The Ghost in Newton's Clockwork Universe*. Plenum Press.
2. Einstein, A. (1915). Erklärung der Perihelbewegung des Merkur aus der allgemeinen Relativitätstheorie. *Sitzungsberichte der Königlich Preußischen Akademie der Wissenschaften*.
3. Grosser, M. (1962). *The Discovery of Neptune*. Harvard University Press.
4. Nieto, M. M. (1972). *The Titius--Bode Law of Planetary Distances: Its History and Theory*. Pergamon Press.
5. Yamagishi, Y. (2026). *The Hidden Symmetry Unique to n=2* [Paper 79]. Zenodo. 10.5281/zenodo.21732820
6. Yamagishi, Y. (2026). *A Broken Law Can Be Blamed on Any of the Assumptions* [Paper 481]. Zenodo. 10.5281/zenodo.22733927

**Yuuki YAMAGISHI (山岸 勇輝)**

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