Tide Tables Reach a Century Ahead Not by Solving the Tide but by Taking It Apart

— fitting an example tide from a **29.53**-day record, the error a century ahead is the same as in the first decade (ratio **0.999937**) — the separator is an outside clock versus an instability growing from within — [Paper 649]

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— *From thinking to syncing.* —

Abstract

Weather is doubtful a week ahead, yet the rise and fall of the tide is sold in tables years in advance. A tide table is not made by solving the flow of the ocean. It splits the tide into waves with periods set by the motions of the Moon and Sun (tidal constituents), fits only their amplitudes and phases to the harbour's record, and adds them.What this procedure fixes, and how far it reaches, is counted.No new theorem or law is claimed. First, spring and neap tides are the beat of two constituents——the semidiurnal M2 (12.420601 hours) and S2 (12.000000 hours) beat with a period of 14.7653 days, matching half the synodic month, 14.7653 days (Section 2). **Second, separating two constituents needs a record about as long as their beat——the difficulty of separating M2 and S2 by fitting (the condition number) was 18.0881 for 1 day of hourly record, 2.5035 for 7 days and 1.4179 for 14.7653 days, falling to near the floor of $\sqrt{2}$** (Section 3). **Third, and this is the core.The error of tide prediction does not grow as the prediction reaches further——an example tide (8 constituents) was fitted with 5 constituents from a 29.53-day record and predicted month by month for a century. The mean error of the last decade was 0.999937 times that of the first. The error came not from time but from the constituents that could not be separated. Fitting all 8 from a 1-year record gave a century maximum of 0.002574 m** (Section 4). **Fourth, the separator is an outside clock versus an instability growing from within——the Moon and Sun that drive the tide keep their periods outside the ocean, and the ocean only responds. Weather grows from an instability within, and, as Paper 289 showed, its error grows day by day** (Section 5). **Keywords**: tide table, harmonic analysis, tidal constituents, spring and neap tides, tide prediction.

Scope of this paper (scope note): **No new theorem or law is claimed——harmonic analysis splitting the tide into a sum of constituents (Doodson 1921 and others) and the need for a long enough record to separate two constituents (a standard rule of tidal analysis) are both known.This is an example——the amplitudes and phases are set as an example and are not the values of any real harbour. The noise is white noise of 0.05 m.What is not treated——the correction for the 18.61-year turning of the Moon's orbital plane, constituents generated in shallow water, and storm surges from typhoons and the like are not included in the example.Relation to earlier papers**: **Paper 289 showed that the limit of forecasting is set by a single number, the rate at which error grows, so that each extra day costs 1.5874 times the initial accuracy——here the side of prediction whose error does not grow is counted, and what allows it is examined.**What is added is **listing the beat periods from the angular speeds of the constituents**, **counting how record length sets the ease of separation (condition number)**, **confirming with an example that the error does not grow over a century**, and **placing the separator on an outside clock versus an instability growing from within**.

1. Notation and the Question

Setting: **the height of the tide at a harbour is written as a sum of constituents.**

$$h(t) = Z_0 + \sum_j A_j \cos(\omega_j t - \varphi_j)$$

The angular speeds ω_j are set by the motions of the Moon and Sun and do not depend on the harbour. The amplitudes A_j and phases φ_j are set by the harbour's shape and the depth of the sea, and are fitted to a record.The periods of the main constituents are M2 (principal lunar semidiurnal) 12.420601 hours, S2 (principal solar semidiurnal) 12.000000 hours, N2 12.658348 hours, K1 23.934470 hours and O1 25.819342 hours.

Question: **what supports a prediction that splits the tide without solving it, and how far does it reach?**

2. Beats

Computation 2.1: **two constituents of nearby periods make a beat. Beat periods were derived from the differences of angular speeds.**

two constituents	beat	what it corresponds to
M2 and S2	**14.7653** days	spring tide to spring tide
K1 and O1	**13.6608** days	strength of diurnal tides
M2 and N2	**27.5546** days	distance of the Moon
K1 and P1	**182.6213** days	half a year
S2 and K2	**182.6211** days	half a year

The M2 and S2 beat of 14.7653 days matched half the synodic month of 29.530589 days, 14.7653 days, within 0.001 days.

> **Observation 2.2**
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> **spring and neap tides are the beat of two constituents.When the peaks of the lunar and solar tides line up, there is a spring tide; when they are apart, a neap tide. The interval at which they line up is set by the period of the apparent positions of the Moon and Sun, so matching half the synodic month follows from how the constituents are built——the strength of the tide comes not from a new cause but from adding periods.**

3. The Length Needed to Separate

Computation 3.1: **the condition number for fitting the amplitudes and phases of M2 and S2 from an hourly record (larger means harder to separate) was derived for each length of record.**

record length	condition number
1 day	18.0881
2 days	8.4393
3 days	5.6471
7 days	2.5035
14.7653 days	1.4179
29.53 days	1.4160
60 days	1.4253

The floor is $\sqrt{2} = 1.414214$, from the ratio of lengths of the constant column and the trigonometric columns.

> **Observation 3.2**
>
> separating two constituents needs a record about as long as their beat.With a record shorter than the beat, the two look like one wave and the fit confuses them. Once the record reaches the beat of 14.7653 days, the condition number fell to near its floor——for the same reason, separating K1 from P1, and S2 from K2, needs half a year of record.

4. The Error a Century Ahead

Computation 4.1: an example tide was built from 8 constituents (M2 1.00, S2 0.45, N2 0.20, K2 0.12, K1 0.40, O1 0.30, P1 0.13, M4 0.03 m) and noise of 0.05 m. Five constituents (M2, S2, N2, K1, O1) were fitted from a 29.53-day record, and the prediction error (root mean square) was derived in 1200 monthly windows over a century.The error ranged from 0.041050 to 0.246136 m, with a first-decade mean of 0.161645 and a last-decade mean of 0.161634, a ratio of 0.999937.

Computation 4.2: fitting all 8 constituents from a 1-year record gave a maximum monthly error over the century of 0.002574 m, with a first-decade mean of 0.002222 and a last-decade mean of 0.002223.

Observation 4.3

this is the core: the error of tide prediction does not grow as the prediction reaches further.With a 29.53-day record, K2 and P1 could not be separated from S2 and K1, and their share became an error that comes and goes with a half-year period. But its size a century later was the same as in the first decade. With a 1-year record and more constituents, the error fell to what remains of the noise——the error of a tide table is set not by how far ahead it looks but by how completely the tide was taken apart.

5. The Separator

Observation 5.1

the separator is an outside clock versus an instability growing from within.The Moon and Sun that drive the tide are outside the ocean, and their periods do not depend on the ocean's state. The ocean is pushed at fixed periods and responds at fixed periods. Weather grows from an instability within the atmosphere, and small differences grow day by day——among things called "prediction", a system answering an outside clock can be tabulated a century ahead, and a system growing from within cannot.

Observation 5.2

beside Paper 289.Paper 289 showed that the limit of forecasting is set by one number, the rate at which error grows, so that each extra day costs 1.5874 times the initial accuracy——in the tide, the thing corresponding to that number does not grow; the ratio of the last decade's error to the first was 0.999937.And at the same harbour, storm surges belong to the weather side, so they do not appear in the tide table.

Observation 5.3

what is not said.The correction for the 18.61-year turning of the Moon's orbital plane, shallow-water constituents and storm surges are not in the example. The amplitudes are not those of a real harbour——what is shown is what supports a prediction that splits and adds.

6. Roll Call and Closing

1. **M2 and S2 beat 14.7653 days** (Computation 2.1): **spring tide to spring tide.**
2. **matches half the synodic month** (Computation 2.1): **follows from the construction.**
3. **K1 and P1 at 182.6213 days** (Computation 2.1): **half a year.**
4. **condition number 18.0881 at 1 day** (Computation 3.1): **cannot separate.**
5. **1.4179 at 14.7653 days** (Computation 3.1): **near the floor $\sqrt{2}$.**
6. **a record as long as the beat** (Observation 3.2): **the length to separate.**
7. **century error ratio 0.999937** (Computation 4.1): **does not grow.**
8. **maximum 0.002574 m with 8 constituents** (Computation 4.2): **falls when separated.**
9. **error set by separation, not time** (Observation 4.3): **the core.**
10. **outside clock or inside instability** (Observation 5.1): **the separator; Paper 289.**

Closing: **A tide table is made not by solving the tide but by splitting it into periods set by the Moon and Sun and adding them.Spring and neap tides are the beat of M2 and S2, 14.7653 days, and separating the two needed a record about that long.Predicting an example tide a century ahead, the error was the same in the first and last decades (ratio 0.999937), and with all constituents separated it fell to a maximum of 0.002574 m——what sets the error is not how far ahead but how completely the tide was taken apart.The separator is an outside clock versus an instability growing from within.Placed among the earlier papers——the tide stands on the side where the limit of Paper 289 does not act.To be honest——the example is not a real harbour, and the 18.61-year correction and storm surges are not included.**

References

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