

The Bekenstein–Hawking Entropy of the Causal Sphere and c^5

— The Dimensional Ladder of Information Capacity —

An Exponent-Structure Observation, Not a Derivation

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— *From thinking to syncing.* —

Positioning of this paper

This paper offers a **structural interpretation**—specifically, an *exponent-structure observation*—of established relations. It does **not** derive the Bekenstein–Hawking entropy, does not claim a literal SI identity $S = \pi c^5$, and does not propose a new physical law. The statement " $S = \pi c^5$ " holds in the c -ladder exponent convention of Papers 44 and 57, where the power of c tracks a dimensional rung, not a dimensionful magnitude. Following the discipline of Paper 48, a numerical/exponent coincidence is reported as a *resonance made precise*, never as an identity of origins. No claim contradicting established theory is made.

ABSTRACT

The dimensional hierarchy of Paper 44 assigns the exponent c^5 to the "capacity" rung (shared with gravitational-wave luminosity and bulk density). This paper gives that rung an *information-theoretic reading*. Combining three established ingredients—the Bekenstein–Hawking entropy $S = A/(4l_P^2)$; the reciprocal-patch scaling $l_P^2 = \hbar G/c^3 \rightarrow c^{-3}$ under $\hbar G = 1$ (Paper 44); and the causal (light) sphere $A = 4\pi c^2$ at $t = 1$ (Papers 60, 61)—we obtain $S = \pi c^5$, with the exponent decomposing exactly as $5 = 2 + 3$ (area c^2 times inverse Planck area c^3). This is the sibling of the Planck-pressure result of Paper 50 (c^7 , $7 = \frac{5}{2} + 3 \cdot \frac{3}{2}$) and is consistent with the c^5 rung of Paper 44. We state this as an **exponent-structure observation**: the " $c^5 = \text{capacity}$ " intuition acquires a Bekenstein–Hawking reading as the information capacity of the causal sphere. We are explicit that this is not a derivation of the entropy law, that $S = \pi c^5$ is an exponent-convention statement (dimensionless in SI), that it depends on the unit-time choice $t = 1$, and that the match of exponents is a resonance, not a shared origin.

1. Introduction

Paper 44 organized the powers of c into a "dimensional ladder," placing c^5 on a rung it labelled *capacity* (the same rung on which the quadrupole gravitational-wave luminosity $L \sim G^{-1}c^5$ and the Planck density sit). That labelling was an exponent-bookkeeping intuition. This paper asks: does the c^5 rung admit an *information-theoretic* reading?

The natural candidate is the Bekenstein–Hawking entropy—the one place in physics where a geometric area is literally a count of information. Using the reciprocal-patch scaling of the Planck length (Paper 44) and the causal-sphere area of Papers 60–61, the Bekenstein–Hawking entropy of the causal sphere lands exactly on the c^5 rung, with a clean exponent decomposition $5 = 2 + 3$.

What this paper does not do. It does not derive the entropy law (that is Bekenstein 1973 / Hawking 1975, cited and assumed). It does not assert $S = \pi c^5$ as a dimensionful SI identity. It reads a rung.

2. The Three Ingredients

2.1 Bekenstein–Hawking entropy (established)

For a horizon (or screen) of area A (with $k_B = 1$):

$$S = \frac{k_B c^3}{4\hbar G} A = \frac{A}{4l_P^2} \quad (l_P^2 = \hbar G/c^3)$$

Taken as given (Bekenstein 1973; Hawking 1975; Jacobson 1995; Padmanabhan 2010).

2.2 Reciprocal-patch scaling of the Planck area (Paper 44)

In the reciprocal patch $\hbar G = 1$ ($\hbar = 1/(4\pi)$, $G = 4\pi$):

$$l_P = \sqrt{\hbar G/c^3} = c^{-3/2}, \quad l_P^2 = c^{-3}$$

Here c is a pure ladder variable (Papers 44, 57): its exponent records a dimensional rung, not an SI magnitude.

2.3 The causal-sphere area (Papers 60, 61)

The light/causal sphere $r = ct$ at unit time $t = 1$ is a Euclidean 2-sphere of radius c :

$$A = 4\pi(ct)^2 \xrightarrow{t=1} 4\pi c^2$$

(4π is the full solid angle of Paper 01; $t = 1$ is the unit-time convention of Paper 61.)

3. The Computation

Proposition 3.1 (Causal-sphere entropy on the c^5 rung)

Substituting 2.2 and 2.3 into 2.1:

$$S = \frac{A}{4l_P^2} = \frac{4\pi c^2}{4c^{-3}} = \pi c^2 \cdot c^3 = \pi c^5 \quad \left(= \pi c^5 / (\hbar G) \right)$$

The exponent decomposes exactly as **5 (entropy) = 2 (area c^2) + 3 (inverse Planck area c^3)**.

Proof. Direct substitution: $A = 4\pi c^2$, $1/l_P^2 = c^3$, hence $S = (4\pi c^2)(c^3)/4 = \pi c^5$. The exponent **5** is the sum of the area exponent **2** and the inverse-Planck-area exponent **3**. ■

Observation 3.2 (Sibling of the Planck-pressure ladder)

The decomposition **5 = 2 + 3** is the entropy analogue of the Planck-pressure result of Paper 50, where the exponent **7** of $P_P = c^7/(4\pi)$ decomposes as **7 = $\frac{5}{2} + 3 \cdot \frac{3}{2}$** . Both are the same ladder arithmetic: a geometric power of c divided by a power of the Planck length. Moreover c^5 is the rung of the quadrupole gravitational-wave luminosity $L \sim G^{-1}c^5$ (Paper 42) and of the Planck density; the causal-sphere entropy joining this rung is a ladder-consistency check, not a new coincidence.

4. Reading the Rung

The content of Proposition 3.1 is a *reading*, not a derivation. In the ordinary reading one holds $\hbar, G, c$ fixed and computes a number. In the ladder reading one tracks the *exponent* of c as a dimensional signature (Papers 44, 57). " $S = \pi c^5$ " is a statement in the second reading: the causal-sphere entropy sits on the fifth rung, built as (two-dimensional area) $\times$ (three inverse Planck-length powers).

This gives the " $c^5 = \text{capacity}$ " intuition of Paper 44 a concrete information-theoretic anchor: up to the π and the $t = 1$ convention, the capacity rung is the Bekenstein–Hawking information content of the causal sphere. That is a satisfying *resonance*. **Per the discipline of Paper 48, we stop there: a shared exponent is not a shared origin.** The c^5 of gravitational-wave luminosity (a radiated power) and the c^5 of causal-sphere entropy (an information count) coincide in ladder position but are not thereby identified.

5. Scope of the Claims

What is established

1. Given the BH law, the reciprocal-patch scaling $l_P^2 = c^{-3}$, and $A = 4\pi c^2$ at $t = 1$, one has $S = \pi c^5$ (Proposition 3.1), with $5 = 2 + 3$.
2. This is the entropy sibling of the Planck-pressure decomposition of Paper 50 and is consistent with the c^5 rung of Paper 44 (Observation 3.2).

What is not established / Open problem

- A derivation of the Bekenstein–Hawking law (it is assumed, not derived).
 - A dimensionful SI identity. " $S = \pi c^5$ " is an exponent-convention statement; in SI, S is dimensionless and the equality is read at the level of the c -exponent.
 - Independence from the unit-time choice (the area, hence S , carries the $t = 1$ convention of Paper 61).
 - A shared origin between the capacity rung and horizon entropy. The exponent match is a resonance (Paper 48 discipline), not an identity.
 - *Open*: is there a covariant object—an entropy density integrated over the causal sphere, independent of $t = 1$ —whose c -exponent is forced to be 5? If so, this reading would be promoted from a rung observation to a covariant statement.
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6. Conclusion

The Bekenstein–Hawking entropy of the causal sphere, evaluated with the reciprocal-patch Planck area $l_P^2 = c^{-3}$ and the causal-sphere area $A = 4\pi c^2$, is $S = \pi c^5$, with the exponent decomposing cleanly as $5 = 2 + 3$. This places the information capacity of the causal sphere on the c^5 rung of the Paper 44 ladder and makes the " c^5 = capacity" intuition precise as an exponent structure. We claim no derivation of the entropy law, no SI identity, and no shared origin with the other c^5 phenomena—only that they occupy the same rung, a resonance recorded under the discipline of Paper 48.

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