The Keel Does Not Drive the Boat but Carries the Side Force, at an Angle That Falls with the Square of the Speed

— the keel is an underwater wing, and the angle of attack (the leeway angle) needed to carry the same side force falls with the square of the speed (**4.000000** times between **3** and **6** knots) — the separator is whether a lifting surface is in the water and water is flowing past it — [Paper 650]

Yuuki YAMAGISHI (山岸 勇輝)
Yamagishi Complexity Torus Lab, Japan
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— *From thinking to syncing.* —

Abstract

On a boat sailing toward the wind, the sail puts most of its force sideways. With nothing resisting that sideways force, the boat would simply be pushed downwind. What takes the sideways force in the water is the keel.Treating the keel as an underwater wing, the angle of attack needed to carry the side force, and how it changes with speed, are counted.No new theorem or law is claimed. First, the keel does not drive the boat——the keel makes only a sideways lift and a drag that resists going forward. The forward part of the sail's force can be used only because the keel takes the sideways part (Section 1). **Second, and this is the core.The angle of attack needed to carry the same side force falls with the square of the speed——with a side force of 1500 N, an area of 1.2 m^2 and an effective aspect ratio of 3, the angle of attack is 8.7541 degrees at 4 knots, 5.6026 degrees at 5 knots and 3.8907 degrees at 6 knots. The ratio between 3 and 6 knots was 4.000000** (Section 2). **Third, when slow, the keel stalls——the speed at which the needed lift coefficient reaches its maximum of 1.0 is 3.0358 knots, and at 3 knots 1.0240 is needed, so the keel stalls. The lift-to-drag ratio peaks at 15.3499 at 5.4790 knots (drag angle 3.7274 degrees) and falls whether faster or slower** (Section 3). **Fourth, the separator is whether a lifting surface is in the water and water is flowing past it ——the angle of attack becomes the boat's leeway angle directly. Heading 40 degrees off the wind, raising the leeway from 5 to 15 degrees cuts the upwind component from 0.707107 to 0.573576, by 18.8840 %** (Section 4). **Keywords**: keel, leeway, lifting-line theory, stall, sailing.

> **Scope of this paper (scope note)**: No new theorem or law is claimed——the lift slope and induced drag of a finite wing (Prandtl's lifting-line theory, 1918), the keel's role in carrying side force, and the leeway of a boat are all known in yacht engineering.This is an example——a side force of 1500 N, a keel area of 1.2 m^2, an effective aspect ratio of 3, a profile drag coefficient of 0.010 and a maximum lift coefficient of 1.0 are set as an example. The sail's side force is held constant, though in reality it moves with the apparent wind.What is not included——the side force of the hull itself, the rudder, and the effects of heel.Relation to earlier papers: Paper 390 showed that a sailing craft can outrun the wind, treating the boat's lift-to-drag ratio as a given number and not entering the mechanism of lift or stall——here, on its underwater side, the lift-to-drag ratio and the angle of attack are counted as they move with speed.Paper 458 showed that raising the lift coefficient lowers the stall speed only by a square root——the keel too has a stall speed of the same form.What is added is listing the angle of attack and lift-to-drag ratio by speed, deriving the speed at which the keel stalls and the speed of the best lift-to-drag ratio, counting how much the angle of attack cuts the upwind component, and placing the separator on an underwater wing and the flow past it.

1. Notation and the Question

Setting: let F be the sideways force made by the sail. The keel is an underwater wing, and at speed V, area A and lift coefficient C_L it makes a sideways lift

$$F = \tfrac{1}{2}\rho V^2 A\, C_L, \qquad C_L = a\,\alpha, \qquad a = \frac{2\pi}{1 + 2/\mathrm{AR}}$$

(α the angle of attack, AR the effective aspect ratio, a the lift slope of lifting-line theory). The drag coefficient is the sum of the profile part and the induced part, $C_D = C_{D0} + C_L^2/(\pi \mathrm{AR})$.The premises are seawater at $\rho = 1025$ kg/m^3, $F = 1500$ N, $A = 1.2$ m^2, $\mathrm{AR} = 3$, $C_{D0} = 0.010$ and a maximum lift coefficient of 1.0.

The keel does not push: the keel makes only a sideways lift and a drag that resists motion; it makes no force pushing forward. When the sail's force is split into forward and sideways parts, the forward part can drive the boat only because the keel takes the sideways part and the boat is not pushed sideways.

Question: how does the angle of attack needed to carry the side force change with speed?

2. Angle of Attack and Speed

Computation 2.1: the lift slope was $a = 3.769911$ per radian. For each speed, the needed lift coefficient, angle of attack and lift-to-drag ratio were derived.

speed	needed lift coefficient	angle of attack	lift-to-drag ratio
2 knots	2.3040	(stalled)	—
3 knots	1.0240	(stalled)	—
4 knots	0.5760	8.7541 degrees	12.7427
5 knots	0.3686	5.6026 degrees	15.0965
6 knots	0.2560	3.8907 degrees	15.1000
7 knots	0.1881	2.8585 degrees	13.6753

Comparing the angles by the formula, the ratio between 3 and 6 knots was 4.000000.

> **Observation 2.2**
>
> this is the core: the angle of attack needed to carry the same side force falls with the square of the speed.Lift goes with the square of the speed, so at half the speed the same side force needs four times the angle——the keel can carry side force only when there is speed.

3. Stall and Lift-to-Drag Ratio

Computation 3.1: the speed at which the needed lift coefficient reaches its maximum of 1.0 was 1.5617 m/s, or 3.0358 knots. At 3 knots, 1.0240 is needed, above the maximum.The lift-to-drag ratio is greatest at a lift coefficient of 0.306998, at a speed of 5.4790 knots, with a maximum of 15.3499 (drag angle 3.7274 degrees). The values 15.0965 at 5 knots and 15.1000 at 6 knots do not exceed it.

> **Observation 3.2**
>
> slow, the keel stalls; too fast, the lift-to-drag ratio falls too.When slow, a large angle is needed, induced drag grows, and at last the keel stalls. When fast, a small angle suffices, but profile drag remains, so the lift-to-drag ratio falls again (13.6753 at 7 knots)——an underwater wing too has a range of speeds in which it works well.

4. Leeway and the Upwind Component

Computation 4.1: **the keel's angle of attack is the difference between where the boat points and where it goes, the leeway angle. Heading 40 degrees off the wind, the upwind component (as a fraction of the boat's speed) was as follows.**

leeway	upwind component
0 degrees	0.766044
5 degrees	0.707107
10 degrees	0.642788
15 degrees	0.573576

Raising the leeway from 5 to 15 degrees cuts the upwind component by 18.8840 %.

> **Observation 4.2**
>
> **a slow boat cannot climb to windward.The angle of 3.8907 degrees at 6 knots is smaller than the 5-degree row, and 8.7541 degrees at 4 knots is close to the 10-degree row. Below 3 knots the keel stalls and the boat is pushed sideways——how close to the wind a boat can sail is set not by the sail alone but by how fast the underwater wing cuts the water.**

5. The Separator

> **Observation 5.1**
>
> **the separator is whether a lifting surface is in the water and water is flowing past it.With no surface there is nothing to carry the side force; with a surface but slow flow, the angle falls short and the keel stalls——what a keel does is set not by its shape alone but by its shape together with speed.**

Observation 5.2

beside Paper 390 and Paper 458.Paper 390 showed that a sailing craft can outrun the wind, treating the boat's lift-to-drag ratio as a given number. Here the underwater wing's lift-to-drag ratio was shown to move with speed, from 12.7427 to 15.3499. Paper 458 showed that raising the lift coefficient lowers the stall speed only by a square root——the keel too has a stall speed of the same form, 3.0358 knots for the boat set here.

Observation 5.3

what is not said.The sail's side force was held constant. The hull's side force, the rudder and the effects of heel are not included. The design of real boats is not discussed——what is shown is how the keel carries side force and how that is bound to speed.

6. Roll Call and Closing

1. **the keel does not push** (Section 1): **it takes the side force.**
2. **lift slope 3.769911 per radian** (Computation 2.1): **aspect ratio 3.**
3. **8.7541 degrees at 4 knots** (Computation 2.1): **large when slow.**
4. **3.8907 degrees at 6 knots** (Computation 2.1): **small when fast.**
5. **ratio 4.000000 between 3 and 6 knots** (Computation 2.1): **inverse square of speed.**
6. **stall speed 3.0358 knots** (Computation 3.1): **cannot carry when slow.**
7. **maximum lift-to-drag ratio 15.3499** (Computation 3.1): **at 5.4790 knots.**
8. **18.8840 % less from 5 to 15 degrees of leeway** (Computation 4.1): **upwind component.**
9. **a slow boat cannot climb** (Observation 4.2): **the speed of cutting water.**
10. **an underwater wing and flow** (Observation 5.1): **the separator; Paper 390 and Paper 458.**

Closing: **The keel does not push the boat. It takes the sideways force of the sail in the water, letting the forward part drive the boat.The angle of attack needed to carry the side force falls with the square of the speed, differing 4.000000 times between 3 and 6 knots.For the boat set here, the keel stalls below 3.0358 knots, and the lift-to-drag ratio is greatest, 15.3499, at 5.4790 knots——and the angle of attack becomes leeway directly, cutting the upwind component.The separator is whether a lifting surface is in the water and water is flowing past it.Placed among the earlier papers——the boat's lift-to-drag ratio, a given number in Paper 390, moves with speed under water.To be honest——this is an example with the sail's side force held constant, and the hull and rudder are not included.**

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Yuuki YAMAGISHI (山岸 勇輝)

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