The Balance of a Swim Bladder Is Unstable: Sink and It Shrinks, So You Sink Further; Rise and It Swells, So You Rise Further

— the absolute pressure at 10 m depth is 1.992037 atmospheres, and a fish balanced at the surface that goes down there finds its bladder 0.501999 times as large — the separator is whether the float shrinks in the same proportion as the fluid around it — [Paper 652]

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— From thinking to syncing. —

Abstract

Many fish make their bodies as heavy as water with a bag of gas in the belly, the swim bladder, and stay at one depth without swimming. But gas shrinks when pressed.Whether that balance is stable or unstable is counted from Boyle's law.No new theorem or law is claimed. First, water presses about one more atmosphere for every 10 m of depth——in seawater the absolute pressure at 10 m is 1.992037 atmospheres, and a fish balanced at the surface that goes down there finds its bladder 0.501999 times as large, losing buoyancy equal to 0.024900 of its weight (Section 2). **Second, and this is the core.The balance of a swim bladder is unstable——a fish balanced at 10 m that sinks 1 m finds its bladder 0.952562 times as large, and the net sinking force is 0.002372 of its weight. The deeper it sinks, the more the bladder shrinks, and the more it shrinks, the deeper it sinks. The displacement grows by a factor *e* in 6.3994 seconds in the fastest case, without drag** (Section 3). **Third, deeper, the instability eases, but the bladder grows expensive——that time is 4.7537 seconds at 1 m, 14.9834 seconds at 100 m and 45.3871 seconds at 1000 m, and the ratio between 1000 m and 10 m matched the square root of the ratio of pressures, 7.092400. Meanwhile, holding the same volume takes 100.203713 times the surface amount of gas at 1000 m** (Section 4). **Fourth, the separator is whether the float shrinks in the same proportion as the fluid around it——gas in water shrinks far more than its surroundings and becomes unstable. A balloon in air keeps a constant lift, because the air thins in the same proportion (Paper 592). A float that does not shrink balances at every depth** (Section 5). **Keywords**: swim bladder, Boyle's law, neutral buoyancy, unstable equilibrium, fish.

Scope of this paper (scope note): **No new theorem or law is claimed——Boyle's law, the pressure of water, and the instability of balance by swim bladder (Alexander 1966 and others) are all known.The fastest case is counted——water drag and added mass (water moving with the body) are not included; with them, the displacement would grow more slowly. The times are those of the fastest growth of the instability.This is an example——seawater at 1025 kg/m^3, a fish of mass 1 kg and a bladder of 5 % of body volume are set, and the gas is taken to shrink at constant temperature.Fish physiology is not discussed——how fish actually correct the displacement, the compressibility of fats, and how deep-sea fish float are not treated.Relation to earlier papers**: **Paper 592 showed that although a rubber balloon swells as it rises, its lift stays constant as long as the amount of gas inside does not change——here, because the surroundings are water rather than air, lift is counted as changing with depth.Paper 393 showed that loading a barge lowers its stability against heeling with the centre of gravity never moved——here restoring against depth rather than heel is examined, and shown to be negative.**What is added is **deriving the time of the instability at each depth**, **setting side by side that this time lengthens as the square root of pressure while the gas needed grows in proportion to pressure**, and **placing the separator on the difference in how the float and its surroundings shrink**.

1. Notation and the Question

Setting: **a fish of mass m balances at depth h_0, as heavy as the water. Its swim bladder has volume V_b, and the gas inside follows Boyle's law: at constant temperature its volume is inversely proportional to pressure. The absolute pressure at depth h is $P(h) = P_0 + \rho g h$.If the fish sinks by δh, the bladder shrinks, buoyancy falls, and a net force toward sinking**

$$F \simeq \frac{(\rho g)^2 V_b}{P(h_0)}\,\delta h$$

acts. The force points the same way as the displacement, so the displacement grows by a factor e every $\tau = \sqrt{m P(h_0)/\{(\rho g)^2 V_b\}}$ (without drag or added mass).The premises are seawater with $\rho = 1025$ kg/m^3, $P_0 = 101325$ Pa, a fish of mass 1 kg, and a bladder of 5 % of body volume.

Question: **is the balance of a swim bladder stable or unstable, and what decides it?**

2. Pressure and Volume

Computation 2.1: **the absolute pressure at each depth, and the factor by which a bladder of surface volume would shrink, were derived.**

depth	absolute pressure	at surface volume
10 m	1.992037 atm	0.501999 times
100 m	10.920371 atm	0.091572 times
1000 m	100.203713 atm	0.009980 times

A fish balanced at the surface that goes down to 10 m finds its bladder 0.501999 times as large, and loses buoyancy equal to 0.024900 of its weight.

3. Instability

Computation 3.1: **a fish balanced at 10 m that sinks 1 m finds its bladder 0.952562 times as large, and the net sinking force is 0.002372 of its weight.**

> **Observation 3.2**
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> **this is the core: the balance of a swim bladder is unstable.Sinking raises the pressure, the bladder shrinks, buoyancy falls, and the fish sinks further. Rising swells the bladder, buoyancy grows, and the fish rises further. The force that appears widens the displacement instead of undoing it——a fish balanced by its swim bladder is not simply at rest at its depth; something must keep correcting it.How fish actually do this is not discussed here.**

4. Depth and the Speed of the Instability

Computation 4.1: **for balance at each depth (the bladder set to 5 % of the body there), the force per metre of displacement and the time for the displacement to grow by a factor e were derived.**

balance depth	force per metre	time to grow by e
1 m	0.044253 N/m	4.7537 s
10 m	0.024419 N/m	6.3994 s
100 m	0.004454 N/m	14.9834 s
1000 m	0.000485 N/m	45.3871 s

The ratio of the times at 1000 m and 10 m was 7.092400, matching the square root of the ratio of pressures, 7.092400.

Computation 4.2: the amount of gas needed to hold the same fraction (5 % of the body), with the surface as 1, was 1.992037 at 10 m, 10.920371 at 100 m and 100.203713 at 1000 m.

> **Observation 4.3**
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> deeper, the instability eases, but the bladder grows expensive.Deep down, the same 1 m of displacement changes the pressure by a smaller fraction, so the time of the instability lengthens as the square root of pressure. But holding the same volume requires packing in more gas in proportion to pressure——the same bag carries a fast instability in shallow water and a heavy upkeep in deep water.

5. The Separator

> **Observation 5.1**
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> the separator is whether the float shrinks in the same proportion as the fluid around it.Water hardly shrinks, so sinking deeper leaves the density of the surroundings unchanged, and only the gas shrinks. For a balloon in air, the gas swells in the same proportion as the air thins with height, so the weight of air it displaces stays the same. A float that does not shrink balances at every depth, even in water——what makes the instability is not the gas itself but the difference between how the gas and its surroundings shrink.

> **Observation 5.2**
>
> beside Paper 592 and Paper 393.Paper 592 showed that although a rubber balloon swells as it rises, its lift stays constant as long as the amount of gas inside does not change. This paper is the mirror side, with the surroundings replaced by water, where lift changes with depth. Paper 393 showed that loading a barge lowers its stability against heeling with the centre of gravity never moved——here restoring against depth, not heel, has been made negative by the swim bladder.A Cartesian diver in a plastic bottle, which sinks when squeezed and rises when released, follows the same formula, and the depth at which it just balances is unstable.

Observation 5.3

what is not said.Drag and added mass are not included, so the times are those of the fastest case. Fish physiology, how fish actually correct, and the compressibility of fats are not treated——what is shown is how the balance of a swim bladder becomes unstable, and how its speed changes with depth.

6. Roll Call and Closing

1. **absolute pressure 1.992037 atm at 10 m** (Computation 2.1): **nearly two atmospheres.**
2. **0.501999 times at surface volume** (Computation 2.1): **losing 0.024900 of buoyancy.**
3. **net 0.002372 after sinking 1 m** (Computation 3.1): **toward sinking.**
4. **a force that widens the displacement** (Observation 3.2): **the core.**
5. **6.3994 seconds at 10 m** (Computation 4.1): **time to grow by *e*.**
6. **45.3871 seconds at 1000 m** (Computation 4.1): **eases with depth.**
7. **ratio 7.092400, the square root of the pressure ratio** (Computation 4.1): **matches.**
8. **100.203713 times the gas at 1000 m** (Computation 4.2): **expensive.**
9. **how float and surroundings shrink** (Observation 5.1): **the separator.**
10. **constant lift of a balloon; heel of a barge** (Observation 5.2): **Paper 592 and Paper 393.**

Closing: **A fish made as heavy as water by its swim bladder is not simply at rest at its depth; it sits on an unstable balance.A fish balanced at 10 m that sinks 1 m feels a net force of 0.002372 of its weight toward sinking, and the displacement grows by a factor *e* every 6.3994 seconds in the fastest case.Deeper, that time lengthens as the square root of pressure (45.3871 seconds at 1000 m), while the gas to be held grows in proportion to pressure (100.203713 times)——sink and it shrinks, so you sink further; rise and it swells, so you rise further.The separator is whether the float shrinks in the same proportion as the fluid around it.Placed among the earlier papers——as the mirror of the balloon in air of Paper 592, gas in water changes its lift with depth.To be honest——this is the fastest case without drag or added mass, and fish physiology is not entered.**

References

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