Choosing a Window Is Choosing the Answer: Lowering the Leakage Loses the Power to Split Close Tones

— a weak tone at −60 dB, 7.68 bins from a strong one, is buried in leakage with the rectangular window (weak over leakage 0.0264) and visible with Blackman (3.3629) — the separator is whether one wants to split close tones or to see a faint neighbour — [Paper 654]

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— From thinking to syncing. —

Abstract

To measure the spectrum of a signal, one can only cut out a record of finite length. Cutting out is multiplying an endless signal by a "window", and the window's shape is convolved into the measured spectrum. Tapering the ends of the window reduces how much a strong tone leaks to distant frequencies, but widens its peak.The trade is counted with three windows.No new theorem or law is claimed. First, the lower a window's leakage, the wider its peak——the highest sidelobe was −13.2632 dB for the rectangular window, −31.4676 for Hann and −58.1088 for Blackman, and the equivalent noise bandwidth 1.0000, 1.5000 and 1.7268 bins (Section 2). **Second, with the rectangular window, a weak tone next to a strong one is buried in leakage——with a 100 Hz tone (amplitude 1) and a 130 Hz tone 7.68 bins away (amplitude 0.001), the weak tone's peak divided by the strong tone's leakage was 0.0264 for the rectangular window, 1.2922 for Hann and 3.3629 for Blackman** (Section 3). **Third, and this is the core.The lower a window's leakage, the less it can split two close tones——adding two equally strong tones at 36 phase differences, they always split into two peaks from 1.5 bins with the rectangular window, from 2.0 bins with Hann and from 2.5 bins with Blackman. At 1.5 bins the share that split was 1.0000, 0.5833 and 0.5278** (Section 4). **Fourth, the separator is whether one wants to split close tones or to see a faint neighbour——no single window answers both questions best at once. Choosing a window is choosing which question to answer** (Section 5).
Keywords: window function, spectral analysis, spectral leakage, frequency resolution, discrete Fourier transform.

> **Scope of this paper (scope note)**: **No new theorem or law is claimed——the sidelobes, main-lobe widths and equivalent noise bandwidths of window functions, and the trade between leakage and resolution, are known in signal processing (Blackman and Tukey 1958, Harris 1978).The optimal window is not discussed——windows that explore the trade continuously, such as the Kaiser or Dolph--Chebyshev windows, are not treated; only three representatives are set side by side.No noise is added——the probability of finding a weak tone in noise is not treated; what is counted is the ratio of a weak tone's peak to the leakage of a strong one.Relation to earlier papers**: **Paper 266 showed that the sampling theorem is correct but its premise (a signal both band-limited and of finite duration) is never met, and stated that it does not treat window functions——this paper takes that seat and counts the price of cutting at a finite length.Paper 520 showed that the number of frequencies that can be nulled is fixed by the number of coefficients, and did not enter methods that apply a window when truncating——here it is examined how such a window divides leakage and resolution.**What is added is **measuring the quantities of three windows directly**, **counting the trade with two experiments, a weak neighbour and two close tones**, **measuring resolution as a share over 36 phase differences**, and **placing the separator on the direction of the question**.

1. Notation and the Question

Setting: $N = 256$ **samples are cut out at a sampling rate of 1000 Hz. One bin (the unit of frequency resolution) is 3.90625 Hz. The cut signal is multiplied by a window $w[n]$ before the Fourier transform.**

$$w_{\mathrm{Hann}}[n] = 0.5 - 0.5\cos\frac{2\pi n}{N}, \qquad w_{\mathrm{Blackman}}[n] = 0.42 - 0.5\cos\frac{2\pi n}{N} + 0.08\cos\frac{4\pi n}{N}$$

The rectangular window is cutting out without multiplying by anything.

Question: **when the window is changed, what is gained and what is lost in the measured spectrum?**

2. The Windows Themselves

Computation 2.1: **the Fourier transform of each window was taken finely (64-fold zero padding), and its quantities measured.**

window	highest sidelobe	-3 dB width	-6 dB width	equivalent noise bandwidth
rectangular	-13.2632 dB	0.9062 bins	1.2188 bins	1.0000 bins
Hann	-31.4676 dB	1.4688 bins	2.0312 bins	1.5000 bins
Blackman	-58.1088 dB	1.6562 bins	2.3125 bins	1.7268 bins

The edge of the main lobe (the first null) was at 1.0000 bins for the rectangular window, 2.0000 for Hann and 3.0000 for Blackman.

> **Observation 2.2**
>
> **the lower a window's leakage, the wider its peak.The sidelobes fall from -13.2632 dB for the rectangular window to -58.1088 dB for Blackman, but the edge of the main lobe widens from 1 bin to 3——tapering the ends reduces leakage far away and blurs what is near.**

3. A Faint Neighbour

Computation 3.1: a strong tone at 100 Hz (amplitude 1) and a weak tone at 130 Hz (amplitude 0.001, -60 dB) were set, 7.68 bins apart. The leakage near 130 Hz with the strong tone alone was compared with the peak of the weak tone alone.

window	leakage of strong tone	peak of weak tone	weak over leakage
rectangular	3.7729×10^{-2}	9.9539×10^{-4}	0.0264
Hann	7.7345×10^{-4}	9.9942×10^{-4}	1.2922
Blackman	2.9723×10^{-4}	9.9955×10^{-4}	3.3629

> **Observation 3.2**
>
> **with the rectangular window, the weak tone is buried in the strong tone's leakage.The weak tone's peak is only 0.0264 times the leakage. With Hann it is about as high as the leakage, and with Blackman 3.3629 times it, visible as a peak——the weak tone was equally present under every window; what decided whether it was seen was how much the strong tone leaked.**

4. Two Close Tones

Computation 4.1: two equally strong tones were added at various separations and 36 phase differences, and the share in which the peak split into two was counted.

separation	rectangular	Hann	Blackman
1.0 bin	0.6111	0.3611	0.3056
1.25 bins	0.8333	0.5000	0.4444
1.5 bins	1.0000	0.5833	0.5278
1.75 bins	1.0000	0.7778	0.6667
2.0 bins	1.0000	1.0000	0.8611
2.5 bins	1.0000	1.0000	1.0000

Observation 4.2

this is the core: the lower a window's leakage, the less it can split two close tones.The tones always split from 1.5 bins with the rectangular window, from 2.0 bins with Hann and from 2.5 bins with Blackman. The window that best revealed the weak tone in Section 3 is the one that splits worst here——from the same record of the same length, changing only the window changes the answer.Measuring the depth of the dip at a single phase difference gave values that moved widely, so the share was counted instead.

5. The Separator

Observation 5.1

the separator is whether one wants to split close tones or to see a faint neighbour.To split close tones, choose the narrow-peaked rectangular window; to see a faint neighbour, choose the low-leakage Blackman. No window satisfies both best at once——choosing a window is choosing what to ask of the spectrum.

Observation 5.2

beside Paper 266 and Paper 520.Paper 266 showed that a signal both band-limited and of finite duration does not exist, so the premise of the sampling theorem is never met, and stated that it does not treat window functions. Paper 520 showed that the number of frequencies that can be nulled is fixed by the number of coefficients, and did not enter applying a window when truncating——this paper counted how the price of finite length comes out split in two directions, leakage and resolution.

Observation 5.3

what is not said.The optimal window is not discussed, and the probability of detection in noise is not treated ——what is shown is that the three windows stand at different points of one and the same trade.

6. Roll Call and Closing

1. **sidelobes -13.2632, -31.4676, -58.1088 dB** (Computation 2.1): **rectangular, Hann, Blackman.**
2. **equivalent noise bandwidths 1.0000, 1.5000, 1.7268** (Computation 2.1): **the peak widens.**
3. **main-lobe edges at 1, 2, 3 bins** (Computation 2.1): **blurs what is near.**
4. **weak over leakage 0.0264 for rectangular** (Computation 3.1): **buried.**
5. **3.3629 for Blackman** (Computation 3.1): **visible.**
6. **leakage decided what was seen** (Observation 3.2): **the weak tone was equally present.**
7. **rectangular splits from 1.5 bins** (Computation 4.1): **high resolution.**
8. **Blackman from 2.5 bins** (Computation 4.1): **low resolution.**
9. **changing only the window changes the answer** (Observation 4.2): **the core.**
10. **what is asked** (Observation 5.1): **the separator; Paper 266 and Paper 520.**

Closing: **Measuring a spectrum means cutting at a finite length, and cutting means applying a window.The sidelobes fell from -13.2632 to -58.1088 dB from rectangular to Blackman, and a -60 dB weak tone next to a strong one was buried in leakage with the rectangular window (0.0264) and seen with Blackman (3.3629).Two equally strong close tones always split at 1.5 bins with the rectangular window, but only at 2.5 bins with Blackman——from the same record, changing only the window changes the answer.The separator is whether one wants to split close tones or to see a faint neighbour.Placed among the earlier papers——in the seat for window functions that Paper 266 and**

Paper 520 declared they would not treat, the price of finite length was counted.To be honest——only three representative windows are compared, and no noise is included.

References

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