Tempering Can Trade Time for Temperature, but Quenching Cannot: Only Changes That Need Diffusion Can Make the Trade

— By the Hollomon--Jaffe equation, tempering steel at **500** °C for **1** hour can be replaced by **477.4060** °C for **4** hours (**22.5940** °C lower) — the separator is whether the change needs diffusion — [Paper 658]

Yuuki YAMAGISHI (山岸 勇輝)
Yamagishi Complexity Torus Lab, Japan
September 2026

— *From thinking to syncing.* —

Abstract

Steel is hardened by cooling it suddenly from high temperature (quenching), then reheated at a moderate temperature to restore toughness (tempering). In the workshop it is said that "if you lower the temperature, just take longer", but that holds only for tempering.The relation between time and temperature is counted for both stages.No new theorem or law is claimed. First, tempering can trade time for temperature——by the Hollomon--Jaffe equation $P = T(20 + \log_{10} t)$, the same tempering as 500 °C for 1 hour is obtained at 477.4060 °C for 4 hours (22.5940 °C lower), 463.1833 °C for 10 hours and 429.7136 °C for 100 hours (Section 2). **Second, and this is the core.The martensite of quenching has no time in it——the fraction of martensite is set only by the temperature to which the steel is cooled. For 0.4 % carbon (Ms 369.8 °C) it is 0.978673 at 20 °C, and for 0.8 % carbon (Ms 200.6 °C) 0.862839 at 20 °C, however many hours one waits there. Cooling to −80 °C raises it to 0.954343** (Section 3). **Third, martensite has no time because diffusion cannot keep up——carbon diffuses 2.4326 μm in 1 second at 900 °C and 0.86482 nm at 300 °C during quenching, a ratio of 2812.8** (Section 4). **Fourth, the separator is whether the change needs diffusion——tempering is a change in which carbon moves and precipitates, so its rate changes with temperature and time can make up for it. Martensite is a change in which atoms only shift, so temperature sets the fraction and time adds nothing** (Section 5). **Keywords**: quenching, tempering, martensite, Hollomon-Jaffe parameter, diffusion.

Scope of this paper (scope note): **No new theorem or law is claimed——the Hollomon--Jaffe equation for tempering (1945), the Koistinen--Marburger equation for the fraction of martensite (1959), Andrews's equation for Ms (1965), and the diffusionless nature of the martensite transformation are all known.The premises are simplified——Ms is taken from the carbon term of Andrews's equation alone, with no alloying elements. The Hollomon--Jaffe constant is set at 20. The diffusion coefficient of carbon in austenite is a representative value ($D_0 = 2.3 \times 10^{-5}$ m^2/s, activation energy 148 kJ/mol), used only at 900 °C and for metastable austenite cooled to 300 °C during quenching.What is not treated——stabilization of retained austenite, quench cracking and hardenability (cooling rate and section size) are not discussed.Relation to earlier papers**: **Paper 562 showed that the upper limit of residual stress in restrained, cooled steel is set by yield, and stated that it does not treat the martensite transformation——this paper takes that seat of phase transformation.Paper 560 showed that a layer grown by diffusion thickens as the square root of time, adding less and less with waiting——here, only changes that need diffusion can trade time for temperature.**What is added is **deriving the equivalent tempering temperature for each time**, **rewriting the trade as an apparent activation energy**, **setting the temperature-only fraction of martensite beside diffusion distances**, and **placing the separator on whether diffusion is needed**.

1. Notation and the Question

Tempering: **the Hollomon--Jaffe equation expresses the progress of tempering as**

$$P = T\,(C + \log_{10} t) \qquad (T \text{ in K},\ t \text{ in hours},\ C = 20)$$

and the same P is taken to give the same hardness.

Quenching: **the Koistinen--Marburger equation expresses the fraction of martensite on cooling to a temperature T below Ms as $f = 1 - \exp\{-0.011(M_s - T)\}$. Ms was taken from the carbon term of Andrews's equation, $M_s = 539 - 423C$ (°C, C in % carbon).**

Question: **in which stage can one say "if you lower the temperature, just take longer"?**

2. The Trade in Tempering

Computation 2.1: **at 500 °C for 1 hour, $P = 15463.00$. The temperature giving the same P was derived for each time.**

time	temperature for the same tempering	lowered by
1 hour	500 °C	—
4 hours	477.4060 °C	22.5940 °C
10 hours	463.1833 °C	36.8167 °C
100 hours	429.7136 °C	70.2864 °C

Computation 2.2: this rate of trade is, at 500 °C, the same as an Arrhenius law with an apparent activation energy of 296.0354 kJ/mol (that Arrhenius law gives 477.4060 °C for 4 hours).

> **Observation 2.3**
>
> tempering can trade time for temperature.Tempering is a change in which carbon trapped in martensite moves and precipitates as carbides, and its rate changes strongly with temperature. What is lost in speed at a lower temperature can be made up with a longer time——a hundred times the time allows a temperature 70.2864 °C lower.

3. Quenching Has No Time in It

Computation 3.1: the fraction of martensite was derived for each temperature reached.

carbon	300 °C	200 °C	100 °C	20 °C	−80 °C
0.4 % (Ms 369.8 °C)	0.535967	0.845537	0.948584	0.978673	0.992901
0.8 % (Ms 200.6 °C)	0.000000	0.006578	0.669319	0.862839	0.954343

> **Observation 3.2**
>
> this is the core: the fraction of martensite is set only by how far the steel is cooled, and has no time in it.Time does not appear in the equation. Steel with 0.8 % carbon held at 20 °C for any number of hours stays at 0.862839; to raise it one can only cool to −80 °C (0.954343)——in quenching, a longer time cannot stand in for a lower temperature.

4. Diffusion Cannot Keep Up

Computation 4.1: **from a representative diffusion coefficient of carbon in austenite, the diffusion distance in 1 second, $\sqrt{Dt}$, was derived: 2.4326×10^{-6} m at $900\ ^\circ\mathrm{C}$ and 8.6482×10^{-10} m at $300\ ^\circ\mathrm{C}$ during quenching, a ratio of 2812.8.**

> **Observation 4.2**
>
> **martensite has no time because it forms before carbon can move.Near Ms, carbon cannot move even 1 nm in 1 second. The change that happens there carries no carbon; the iron lattice only shifts, and whether it proceeds is set by temperature——quenching must be fast so that other changes that need diffusion (such as pearlite) have no time to happen.**

5. The Separator

> **Observation 5.1**
>
> **the separator is whether the change needs diffusion.A change that needs diffusion has a rate set by temperature, so time and temperature can be traded. A change that needs no diffusion has an extent set by temperature, so time adds nothing——of the two stages of one heat treatment, one has time in it and the other does not.**

> **Observation 5.2**
>
> **beside Paper 562 and Paper 560.Paper 562 showed that the upper limit of residual stress in restrained, cooled steel is set by yield, and stated that it does not treat the martensite transformation. Paper 560 showed that a layer grown by diffusion thickens as the square root of time——here it was counted that only the diffusion side (tempering) can trade time for temperature, while the diffusionless change (quenching) is set by temperature alone.**

Observation 5.3

what is not said.Alloying elements, stabilization of retained austenite, quench cracking and hardenability are not treated. The diffusion coefficient is a representative value——what is shown is what separates the stage where trading time for temperature works from the stage where it does not.

6. Roll Call and Closing

1. **22.5940 °C lower for 4 hours** (Computation 2.1): **the trade in tempering.**
2. **70.2864 °C lower for 100 hours** (Computation 2.1): **time makes up for it.**
3. **apparent activation energy 296.0354 kJ/mol** (Computation 2.2): **the same as Arrhenius.**
4. **0.978673 at 20 °C for 0.4 % carbon** (Computation 3.1): **set by temperature alone.**
5. **0.862839 for 0.8 % carbon** (Computation 3.1): **waiting adds none.**
6. **0.954343 at −80 °C** (Computation 3.1): **only colder helps.**
7. **quenching has no time in it** (Observation 3.2): **the core.**
8. **diffusion distance ratio 2812.8 between 900 and 300 °C** (Computation 4.1): **diffusion cannot keep up.**
9. **speed keeps other changes from happening** (Observation 4.2): **pearlite.**
10. **whether diffusion is needed** (Observation 5.1): **the separator; Paper 562 and Paper 560.**

Closing: **“If you lower the temperature, just take longer” holds for tempering but not for quenching.Tempering at 500 °C for 1 hour was matched by 4 hours at 22.5940 °C lower, the same trade as an Arrhenius law with an apparent activation energy of 296.0354 kJ/mol.The fraction of martensite from quenching was set only by the temperature reached (0.978673 at 20 °C for 0.4 % carbon), and waiting did not raise it; near Ms, carbon moves only 0.86482 nm in 1 second——only changes carried by diffusion can trade time for temperature.The separator is whether the change needs diffusion.Placed among the earlier papers——in the seat of phase transformation left by Paper 562, the stage that has time in it was separated from the stage that does not.To be honest——Ms was taken from the carbon term alone, and alloying elements and hardenability are not included.**

References

1. Andrews, K. W. (1965). Empirical formulae for the calculation of some transformation temperatures. *Journal of the Iron and Steel Institute*.
2. Hollomon, J. H., & Jaffe, L. D. (1945). Time-temperature relations in tempering steel. *Transactions of the American Institute of Mining and Metallurgical Engineers*.

3. Koistinen, D. P., & Marburger, R. E. (1959). A general equation prescribing the extent of the austenite-martensite transformation in pure iron-carbon alloys and plain carbon steels. *Acta Metallurgica*.

4. Yamagishi, Y. (2026). *The Fastest Moment Is the First, and Waiting Longer Adds Almost Nothing* [Paper 560]. Zenodo. 10.5281/zenodo.23014242

5. Yamagishi, Y. (2026). *What Caps a Residual Stress Is Not the Temperature but the Yield* [Paper 562]. Zenodo. 10.5281/zenodo.23014250

— From thinking to syncing. —

Yuuki YAMAGISHI (山岸 勇輝)

© 2026 Yamagishi Complexity Torus Lab, Japan · Licensed under Creative Commons Attribution 4.0 International (CC BY 4.0)