Doubling a Window Does Not Double the Light at a Point: What Counts Is the Cosine-Weighted View of the Sky

— widening a window's half-angle from **45** to **90** degrees multiplies the solid angle by **3.414214** but the light by only **2.000000**, and when the window's radius equals its distance, doubling its area gives only **1.333333** times the light — the separator is window area versus the view of the sky — [Paper 662]

Yuuki YAMAGISHI (山岸 勇輝)
Yamagishi Complexity Torus Lab, Japan
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— From thinking to syncing. —

Abstract

It is natural to think that a larger window makes a room brighter. But the brightness at a particular point in a room is set not by the window's area but by how much of the sky visible from that point lies in directions close to straight ahead.Under a uniformly bright sky, how window size and an opposite building act on the brightness at a point is counted.No new theorem or law is claimed. First, the brightness at a point is set not by solid angle but by the cosine-weighted view——widening a round window's half-angle from 10 to 20 degrees multiplies the solid angle by 3.969616 and the brightness by 3.879385, nearly the same, but from 45 to 90 degrees the solid angle rises 3.414214 times while the brightness rises only 2.000000 times (Section 2). **Second, and this is the core.Near a window, doubling the window does not double the brightness——with the window's radius over distance at 0.1, doubling the area gives 1.980392 times the brightness, but at 1.0 only 1.333333 times, and at 2.0 1.111111 times. The view saturates at its upper limit π** (Section 2). **Third, an opposite building takes sky without changing the window's area ——when an opposite building hides the sky up to elevation θ, the sky light reaching a wall window becomes $1-(2\theta+\sin 2\theta)/\pi$ of its unobstructed value: 0.391002 at 30 degrees, 0.181690 at 45 and 0.057669 at 60. With a 30 m building 20 m away, a window 1.5 m up received 0.090211 and one 29 m up 0.936444** (Section 3). **Fourth, the separator is window area versus the view of the sky——the total light entering through a window grows in proportion to its area, but the brightness at a point is set by the view of the sky, saturates, and is cut away by an opposite building** (Section 4). **Keywords**: daylighting, solid angle, cosine law, sky view, opposite buildings.

Scope of this paper (scope note): **No new theorem or law is claimed——that light reaching a surface is proportional to the cosine of incidence (Lambert's cosine law), that the illuminance from a uniform sky is set by the cosine-weighted solid angle, and that surrounding buildings reduce daylight are all known in daylighting (Hopkinson and colleagues 1966 and others).The sky is taken as uniformly bright——the luminance distribution of the sky, direct sunlight, reflection from the ground or the opposite building, glass transmittance and reflection inside the room are not included.The geometry is simplified——the window is a round window seen face-on, and the opposite building is infinitely long and parallel to the window.Relation to earlier papers**: **Paper 587 showed that under a uniform sky a horizontal skylight receives 2.000000 times the light of a wall window of the same size, and stated that it did not treat surrounding buildings or the brightness inside the room (the daylight factor)——this paper takes that seat and counts, with the same cosine weighting, how window size and an opposite building act on the brightness at a point.**What is added is **setting solid angle beside the cosine-weighted view to show saturation**, **deriving by distance how much brightness doubling the window's area gives**, **deriving by closed form and numerical integration the share of sky hidden by an opposite building**, and **placing the separator on area versus view**.

1. Notation and the Question

Setting: **when the sky's luminance L is uniform, the illuminance at a point on a surface is**

$$E = L \int \cos\theta \, d\Omega$$

where θ is measured from the surface's normal. The integral is called the "cosine-weighted view". For the whole hemisphere of sky it is π, the upper limit.For a round window of half-angle α seen face-on, the solid angle is $2\pi(1-\cos\alpha)$ and the cosine-weighted view $\pi \sin^2\alpha$.With an infinitely long building in front of a wall window hiding the sky below elevation θ_0, the sky light reaching the window is $1-(2\theta_0+\sin 2\theta_0)/\pi$ of its unobstructed value.

Question: **is the brightness at a point set by the window's area or by the view of the sky?**

2. Window Size

Computation 2.1: **the solid angle and cosine-weighted view of a round window of half-angle α were derived.**

half-angle	solid angle	cosine-weighted view	fraction of π
10 degrees	0.095456 sr	0.094731	0.030154
20 degrees	0.378922 sr	0.367497	0.116978
30 degrees	0.841787 sr	0.785398	0.250000
45 degrees	1.840302 sr	1.570796	0.500000
60 degrees	3.141593 sr	2.356194	0.750000
90 degrees	6.283185 sr	3.141593	1.000000

From 10 to 20 degrees, the solid angle rose 3.969616 times and the brightness 3.879385 times. From 45 to 90 degrees, the solid angle rose 3.414214 times and the brightness 2.000000 times.

Computation 2.2: the factor by which the brightness at the point straight ahead rises when the window's area is doubled (radius times $\sqrt{2}$) was derived: 1.980392 with radius over distance 0.1, 1.666667 at 0.5, 1.333333 at 1.0 and 1.111111 at 2.0.

> **Observation 2.3**
>
> this is the core: near a window, doubling the window does not double the brightness.For a small or distant window, the view is nearly proportional to area. But the larger the window appears, the more the added sky lies off to the side, lighting the surface obliquely, and the cosine weighting reduces its effect——the view saturates at its upper limit π, and beyond that, widening the window does not brighten the point.

3. The Opposite Building

Computation 3.1: with an opposite building hiding the sky up to elevation θ_0, the sky light reaching a wall window (with the unobstructed case as 1) was derived by closed form and by numerical integration.

hidden up to	closed form	numerical integration
10 degrees	0.780020	0.780020
20 degrees	0.573172	0.573172
30 degrees	0.391002	0.391002
45 degrees	0.181690	0.181690
60 degrees	0.057669	0.057669

Computation 3.2: with a 30 m building 20 m away, the hidden elevation and the sky light reaching the window were derived for each window height.

window height	hidden up to	sky light reaching
1.5 m	54.9406 degrees	0.090211
10 m	45.0000 degrees	0.181690
20 m	26.5651 degrees	0.450185
29 m	2.8624 degrees	0.936444

> **Observation 3.3**
>
> an opposite building takes sky without changing the window's area.What is hidden is the low sky near the horizon, but for a wall window that is the sky nearest straight ahead, with the largest cosine weight. Hidden up to 45 degrees, the sky light reaching the window falls to 0.181690——within one building, a low window of the same area receives 0.090211 against 0.936444 for a high one.

4. The Separator

Observation 4.1

the separator is window area versus the view of the sky.The total light entering a room through a window is the illuminance on the window's surface times its area, and grows in proportion to area. But the brightness at a point in the room is set by the cosine-weighted view of the sky from that point; it saturates as the window appears larger, and is cut away by an opposite building——"a bigger window is brighter" is right for the total and increasingly wrong, the closer one is, for the brightness at a point.

Observation 4.2

beside Paper 587.Paper 587 showed that under a uniform sky a skylight receives 2.000000 times the light of a wall window. That is because the view of a horizontal surface is π and of a vertical one $\pi/2$, the same cosine weighting——here, with that weighting, window size and the effect of the surrounding buildings that Paper 587 did not treat were counted.

Observation 4.3

what is not said.The sky's luminance is taken as uniform, and direct sunlight, reflections, glass transmittance and reflection inside the room are not included. Daylight-factor standards are not discussed ——what is shown is that the brightness at a point is set not by area but by view.

5. Roll Call and Closing

1. **3.879385 times from 10 to 20 degrees** (Computation 2.1): **near area for small windows.**
2. **solid angle 3.414214 times, brightness 2.000000 from 45 to 90** (Computation 2.1): **the cosine weighting.**
3. **upper limit π** (Computation 2.1): **the whole hemisphere.**
4. **doubling area at radius over distance 1.0 gives 1.333333** (Computation 2.2): **less effect when near.**
5. **1.980392 at 0.1** (Computation 2.2): **nearly double when far.**
6. **the view saturates** (Observation 2.3): **the core.**
7. **0.181690 when hidden up to 45 degrees** (Computation 3.1): **closed form and integration agree.**

8. **0.090211 at 1.5 m, 0.936444 at 29 m** (Computation 3.2): **differs by floor.**
9. **the building takes sky without changing area** (Observation 3.3): **the sky nearest straight ahead.**
10. **area or view** (Observation 4.1): **the separator; Paper 587.**

Closing: **A larger window lets more total light into a room, but a point in the room does not brighten in proportion to the window's area.The brightness at a point is set by the cosine-weighted view of the sky: widening the half-angle from 45 to 90 degrees gave only 2.000000 times, and doubling the area of a window whose radius equals its distance gave only 1.333333 times.An opposite building hiding the sky up to 45 degrees cut the sky light reaching a wall window to 0.181690, and in the same building a low window received only 0.090211——what sets the brightness is not the window's area but the view of the sky.The separator is window area versus the view of the sky.Placed among the earlier papers——with the same cosine weighting as Paper 587, window size and surrounding buildings, which Paper 587 did not treat, were counted.To be honest——the sky is taken as uniform, and reflections and room interreflection are not included.**

References

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Yuuki YAMAGISHI (山岸 勇輝)

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