Condensation Happens Without Any Attraction: What Decides Is the Wavelength Overtaking the Spacing

— the condensation temperature of an ideal Bose gas contains only constants, the mass and the number density, with no interparticle potential at all — the separator is whether switching off the interaction makes it disappear — [Paper 663]

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— From thinking to syncing. —

Abstract

It is natural to think that a phase changes because particles attract one another. But the condensation of bosons happens without any attraction at all.The condensation temperature of an ideal Bose gas is computed in three cases, and what fixes that temperature is examined.No new theorem or law is claimed. First, no interparticle potential appears in the formula for the condensation temperature——only the Planck constant, the Boltzmann constant, the mass of the particle and the number density enter it; no quantity measuring the strength of an interaction appears (Section 2). **Second, the temperature is nevertheless definite——Rb87 at 10^{20} per cubic metre gives $\mathbf{398.3307}$ nK, and at the density of liquid helium (2.181607×10^{28} particles) $\mathbf{3.134378}$ K, the latter being $\mathbf{1.439902}$ times the measured lambda point of 2.1768 K** (Section 2). **Third, and this is the core. The condition is a single dimensionless number——at the condensation temperature, the number density times the cube of the thermal de Broglie wavelength is exactly $\mathbf{2.6123753487}$, agreeing to 10 digits across three cases of different mass and density** (Section 3). **Fourth, the boundary is where the wavelength overtakes the spacing——for Rb87 the mean spacing is 2.154435×10^{-7} m while the thermal de Broglie wavelength at the condensation temperature is 2.967189×10^{-7} m, or $\mathbf{1.377247}$ times as long** (Section 3). **Fifth, raising the density does not raise the temperature in step——ten times the density gives only $\mathbf{4.641589}$ times the temperature, which is ten to the two thirds** (Section 2). **Sixth, the separator is whether switching off the interaction makes it disappear——the computation here contains no interaction and still condenses, so switching one off changes nothing. A condensation that requires pairs to form does disappear** (Section 4). **Keywords**: Bose-Einstein condensation, ideal Bose gas, phase-space density, thermal de Broglie wavelength, critical temperature, Bose statistics.

Scope of this paper (scope note): **No new theorem or law is claimed——condensation of the ideal Bose gas (Einstein 1925), the thermal de Broglie wavelength, and the value of the Riemann zeta function at $3/2$ are all standard.Interacting condensates are not treated——scattering lengths, mean-field equations and the shift of the condensation temperature by interactions are not included. Only the ideal gas is examined.Experiments are not discussed——how the gas is cooled (evaporative cooling, magneto-optical traps) and how the condensate is observed are not the subject.Superconductivity is not explained——condensation that requires pairing is not entered into. The separator "does it disappear when the interaction is switched off" is placed, and no more.Finite systems are not treated——the counting is in the thermodynamic limit, and the rounding that comes with small particle numbers is not included.Relation to earlier papers**: **Paper 482 showed that the words "critical dimension" name two different numbers above and below, and stated that it did not treat condensation itself, the existence proof of a phase transition, behaviour in finite systems, or the interacting case——this paper takes that seat and counts, for the same ideal Bose gas, the condition for condensation.**What is added is **showing from the contents of the formula that no interaction enters the condensation temperature**, **confirming to 10 digits that three different cases return the same phase-space density**, **deriving the ratio of wavelength to mean spacing**, and **placing the separator on switching off the interaction**.

1. Notation and the Question

Setting: **bosons of mass m at number density n exert no force on one another. Their thermal de Broglie wavelength at temperature T is λ.**

$$\lambda = \frac{h}{\sqrt{2\pi m k_B T}}, \qquad T_c = \frac{2\pi\hbar^2}{m k_B}\left(\frac{n}{\zeta(3/2)}\right)^{2/3}$$

Here $\zeta(3/2) = 2.6123753487$.What appears in these two formulas is constants, the mass and the number density.

Question: **is condensation caused by attraction between particles, or by the way they are counted?**

2. A Temperature That Comes Out of Statistics Alone

Computation 2.1: **the condensation temperature was computed in three cases.**

case	number density ($1/m^3$)	condensation temperature
dilute Rb87	1.000000×10^{20}	**398.3307** nK
Rb87 at ten times the density	1.000000×10^{21}	**1.848887** μK
at the density of liquid helium	2.181607×10^{28}	**3.134378** K

The 3.134378 K obtained at the density of liquid helium is 1.439902 times the measured lambda point of 2.1768 K. A value counted for an ideal gas lands in the same order as the transition of the real, interacting liquid.

Computation 2.2: ten times the density raised the condensation temperature by 4.641589, which is ten to the two thirds, as the exponent in the formula requires.

> **Observation 2.3**
>
> it was not an interaction that fixed this temperature.The quantities used above are four: the Planck constant, the Boltzmann constant, the mass and the number density. No strength of attraction, no scattering length and no shape of a potential was used once——and still condensation begins at a finite temperature.

3. The Condition Is One Dimensionless Number

Computation 3.1: the phase-space density (number density times the cube of the thermal de Broglie wavelength) at the condensation temperature was computed in the three cases.

case	phase-space density at the condensation temperature
dilute Rb87	2.6123753487
Rb87 at ten times the density	2.6123753487
at the density of liquid helium	2.6123753487

Three cases whose masses differ by 21.7 times and whose densities differ by 2.2×10^{8} times return the same 2.6123753487, which is $\zeta(3/2)$ itself.

Computation 3.2: for Rb87 at $n = 10^{20}$, the mean spacing is 2.154435×10^{-7} m and the thermal de Broglie wavelength at the condensation temperature is 2.967189×10^{-7} m, a ratio of 1.377247. At $1\ \mu$K the phase-space density is 0.656752, below one.

Observation 3.3

the boundary lies where the wavelength overtakes the spacing.Cooling stretches the thermal de Broglie wavelength until it catches the distance between particles. Once it has, nothing keeps the particles from occupying the same state——the condition for condensation is not “is the attraction strong enough” but “has the wavelength overtaken the spacing”.

Computation 3.4: **the condensed fraction is $1-(T/T_c)^{3/2}$: 0.146185 at $T/T_c = 0.9$, 0.646447 at 0.5 and 0.968377 at 0.1.**

4. The Separator

Observation 4.1

the separator is whether switching off the interaction makes it disappear.The computation here contains no interaction, so switching one off leaves this condensation untouched——a condensation that requires pairs to form does disappear. The one word “condensation” divides in two at exactly this point.

Observation 4.2

beside Paper 482.Paper 482 looked at the infrared side of the integral for an ideal Bose gas, showed that the words “critical dimension” name two different numbers above and below, and stated that condensation itself was not its subject——in that seat, this paper counts the condition for condensation of the same ideal gas.

Observation 4.3

what is not said.Interacting condensates, methods of cooling, and the rounding of finite particle numbers are not treated. That the lambda point of liquid helium differs by a factor of 1.439902 from the ideal-gas value is evidence that interactions act, and the size of that effect is not explained here——what is shown is that the condition for condensation stands without any attraction.

5. Roll Call and Closing

1. **only four quantities enter** (Observation 2.3): **no potential.**
2. **398.3307 nK for Rb87** (Computation 2.1): **a dilute gas.**
3. **3.134378 K at helium's density** (Computation 2.1): **1.439902 times the lambda point.**
4. **ten times the density gives 4.641589** (Computation 2.2): **ten to the two thirds.**
5. **phase-space density 2.6123753487** (Computation 3.1): **ten digits, three cases.**
6. **wavelength 1.377247 times the spacing** (Computation 3.2): **the boundary.**
7. **0.656752 at 1 μK** (Computation 3.2): **not yet there.**
8. **0.646447 at $T/T_c = 0.5$** (Computation 3.4): **the condensed fraction.**
9. **switching it off changes nothing** (Observation 4.1): **the separator.**
10. **the seat of Paper 482** (Observation 4.2): **the other side of the same ideal gas.**

Closing: **Condensation does not happen because particles attract one another.The formula for the condensation temperature of an ideal Bose gas contains only constants, the mass and the number density, and it still returned definite temperatures: 398.3307 nK for Rb87 and 3.134378 K at the density of liquid helium.At those temperatures, three cases of widely different mass and density returned the same phase-space density, 2.6123753487——the condition is not the strength of an attraction but the thermal de Broglie wavelength overtaking the spacing between particles, by a factor of 1.377247 in the example here.The separator is whether switching off the interaction makes it disappear.Placed among the earlier papers——in the seat Paper 482 left open by not treating condensation itself, the condition for it was counted.To be honest——the factor of 1.439902 against the lambda point is the work of interactions, and its size is not explained here.**

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