Rock Convects Not Because It Is Soft but Because It Is Large

— Even at a viscosity of 10^{21} Pa·s, a solid by any human measure, the mantle's Rayleigh number is 6.457708×10^{7}, or 3.780860×10^{4} times the critical value of 1708 — the separator is viscosity against the cube of the thickness — [Paper 669]

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— *From thinking to syncing.* —

Abstract

Rock is a solid. Told that this solid flows and convects, one is tempted to conclude that it must be remarkably soft.But what decides whether it convects is not the viscosity; it is the cube of the thickness. That is shown in numbers.No new theorem or law is claimed. First, even at solid viscosities the critical value is passed by far——for a mantle of viscosity 10^{21} Pa·s, thickness 2.9×10^{6} m and temperature difference 2000 K, the Rayleigh number is 6.457708×10^{7}, or 3.780860×10^{4} times the critical value of 1708 (Section 2). **Second, a thousand times the viscosity is not enough——at 10^{24} Pa·s the Rayleigh number falls to 6.457708×10^{4}, still 37.80860 times critical, and it still convects** (Section 2). **Third, and this is the core. What acts is the cube of the thickness——with the material and the temperature difference fixed, the Moon (1.3×10^{6} m) gives 3.405859×10^{3} times critical, Mars (1.7×10^{6} m) 7.616288×10^{3} and the Earth 3.780860×10^{4}, the Earth-to-Moon ratio of 11.101047 agreeing to 10 digits with the cube of the thickness ratio** (Section 3). **Fourth, to stop, a layer must be thin——at this viscosity and temperature difference, the least thickness at which convection begins is 86.4041 km** (Section 3). **Fifth, the same viscosity appears as a time in another observation——at the same 10^{21} Pa·s, the relaxation time of a load of wavelength 2×10^{6} m is 6152.3536 years, the same formula and the same value as in Paper 433** (Section 4). **Sixth, the separator is viscosity against the cube of the thickness——a difference that a thousandfold viscosity could not make is made by halving the thickness** (Section 4). **Keywords**: mantle convection, Rayleigh number, critical Rayleigh number, viscosity, thermal convection, Earth's interior.

Scope of this paper (scope note): **No new theorem or law is claimed——the Rayleigh number, the critical Rayleigh number, the existence of mantle convection and the representative viscosity are all standard (Turcotte and Schubert 2014, Chandrasekhar 1961).The critical value is not solved for——the critical Rayleigh number 1708 is a literature value for rigid boundaries above and below; no linear stability calculation is carried out here, and other boundaries give other values.The real mantle is not modelled——temperature profiles, phase transitions, the depth and temperature dependence of viscosity and compositional differences are not included. A uniform layer is assumed.Plate motion is not discussed——the relation between convection and surface plates, and the division of driving forces, are not the subject.Representative values are used——density, thermal expansivity, temperature difference, thermal diffusivity and thickness are representative, not the values of a particular model.Relation to earlier papers**: **Paper 645 compared, for a thin liquid layer heated from below, whether buoyancy or the difference in surface tension starts the motion, and put the crossover at 25.4358 mm——this paper uses the same critical Rayleigh number and asks the same question at 10^9 times the thickness.Paper 433 treated post-glacial rebound at the same mantle viscosity of 10^{21} Pa·s——one and the same number appears there as a relaxation time and here as whether the layer convects.**What is added is **computing the Rayleigh number at solid viscosities and setting it against the critical value**, **showing that a thousandfold viscosity still passes it**, **putting the Moon, Mars and the Earth through one formula to confirm the cube**, **inverting for the least thickness at which convection begins**, and **placing the separator on viscosity against the cube of the thickness**.

1. Notation and the Question

Setting: **a layer of thickness L is heated from below with a temperature difference ΔT across it. The strength of buoyancy is measured by the Rayleigh number.**

$$Ra = \frac{\rho g \alpha \, \Delta T \, L^3}{\eta \, \kappa}$$

Here ρ is the density, α the thermal expansivity, η the viscosity and κ the thermal diffusivity. Above a critical value, heat is carried by flow rather than by conduction.The critical value for rigid boundaries, 1708, is taken as given.

Question: **does rock convect because its viscosity is smaller than one thought, or because the layer is thick?**

2. Even at Solid Viscosities, Far Above Critical

Computation 2.1: **with representative mantle values (density 4500 kg/m^3, expansivity 3×10^{-5} per K, temperature difference 2000 K, diffusivity 10^{-6} m^2/s, thickness 2.9×10^6 m, viscosity 10^{21} Pa·s), the Rayleigh number is 6.457708×10^7, which is 3.780860×10^4 times the critical value of 1708.**

Computation 2.2: raising only the viscosity a thousandfold, to 10^{24} Pa·s, gives a Rayleigh number of 6.457708×10^{4}, still 37.80860 times critical.

> **Observation 2.3**
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> softness is not the answer.A viscosity of 10^{21} Pa·s is 10^{24} times that of water; on human timescales there is no word for it but solid. And still it is forty thousand times over critical——since revising the viscosity by a factor of a thousand does not change the conclusion, the reason for convection cannot be sought in the viscosity.

3. What Acts Is the Cube of the Thickness

Computation 3.1: with the material and the temperature difference fixed, only the thickness of the layer was changed.

	thickness	Rayleigh number	times critical
Moon	1.30×10^{6} m	5.817207×10^{6}	3.405859×10^{3}
Mars	1.70×10^{6} m	1.300862×10^{7}	7.616288×10^{3}
Earth	2.90×10^{6} m	6.457708×10^{7}	3.780860×10^{4}

The Earth-to-Moon ratio is 11.101047, agreeing to 10 digits with the cube of the thickness ratio. The exponent in the formula shows through directly.

Computation 3.2: inverting, the least thickness at which convection begins at this viscosity and temperature difference is 86.4041 km.

> **Observation 3.3**
>
> this is the core: size decides the regime.A critical value that a thousandfold viscosity could not bring the layer below is passed downward simply by making it thinner than 86.4041 km——the same rock flows inside a large body and stands still inside a small one. What decides is not the material but the dimension.

Computation 3.4: the estimate for how much heat is carried (the Nusselt number) is 80.2397, eighty times what conduction alone would carry, and the estimated flow speed is 1.504147×10^{-8} m/s, that is 47.4673 cm per year.

4. The Separator

Observation 4.1

the separator is viscosity against the cube of the thickness.A thousandfold viscosity does not bring the layer below critical; reducing the thickness from 2.9×10^{6} m to 86.4041 km does——which of the two is acting can be found by moving them separately.

Observation 4.2

beside Paper 645.Paper 645 compared, in a liquid layer of a few tens of millimetres, whether buoyancy or surface tension starts the motion, using the same notion of a critical value——this paper asks the same question at 10^{8} times the thickness. Because thickness enters as a cube, buoyancy overwhelms in a large system.

Observation 4.3

beside Paper 433.Paper 433 treated the rebound after the ice was gone and, with the same 10^{21} Pa·s, put the relaxation time of a load of wavelength 2×10^{6} m at 6152.3536 years——one viscosity becomes, at the surface, a time in which the land has still not finished rising, and inside, a regime in which the layer convects.

Observation 4.4

what is not said.Temperature profiles, phase transitions and depth-dependent viscosity are not included, and the critical value is simply taken from the literature——what is shown is that whether a layer convects is decided not by viscosity but by the cube of its thickness.

5. Roll Call and Closing

1. $Ra = 6.457708 \times 10^{7}$ (Computation 2.1): **at solid viscosity.**
2. $\mathbf{3.780860} \times 10^{4}$ **times critical** (Computation 2.1): **forty thousand.**
3. **37.80860 times at a thousandfold viscosity** (Computation 2.2): **still over.**

4. **3.405859×10^{3} for the Moon** (Computation 3.1): **same material, other thickness.**
5. **ratio 11.101047** (Computation 3.1): **the cube, to 10 digits.**
6. **least thickness 86.4041 km** (Computation 3.2): **the core.**
7. **Nusselt number 80.2397** (Computation 3.4): **eighty times conduction.**
8. **47.4673 cm per year** (Computation 3.4): **an estimate.**
9. **relaxation time 6152.3536 years** (Observation 4.3): **same viscosity; Paper 433.**
10. **viscosity or thickness** (Observation 4.1): **the separator; Paper 645.**

Closing: **Rock flows, and not because rock is soft.At 10^{21} Pa·s the Rayleigh number is 6.457708×10^{7}, 3.780860×10^{4} times critical, and a thousandfold viscosity still leaves it 37.80860 times over.Change only the thickness to the Moon's and it falls to 3.405859×10^{3} times, the Earth-to-Moon ratio 11.101047 matching the cube of the thickness ratio to 10 digits; convection begins only above 86.4041 km——the same rock flows in a large body and stands still in a small one.The separator is viscosity against the cube of the thickness.Placed among the earlier papers——the critical value Paper 645 used in a layer of millimetres was used here at 10^{8} times the thickness, and the viscosity of Paper 433 appears as 6152.3536 years in another observation.To be honest——a uniform layer was assumed and the critical value was taken from the literature.**

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