What Is Frightening About Exponential Growth Is Not Its Speed but That Half of It Sits in the Last Doubling

— At a specific growth rate of **0.4** per hour the doubling time is **1.732868** hours, and with **0.1** g/L of cells, **20** g/L of substrate and a yield of **0.5** the substrate lasts **11.537801** hours — the separator is whether the limit is one's own capacity to grow or what is left of the resource — [Paper 680]

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— From thinking to syncing. —

Abstract

Exponential growth is explained as being fast. But the same relation says something else: half the consumption falls in the last doubling, which is a statement about how abruptly it stops.Why watching does not help is counted here, in the division of time and substrate.No new theorem or law is claimed. First, and this is the core. The last doubling consumes half the substrate——of 11.537801 hours in all, the final 1.732868 hours, that is 0.150190 of the time, consumes 0.505000 of the substrate (Section 2). **Second, turned around, almost nothing appears to happen for a long time——the first 9.804933 hours consume 0.495000, and most of even that falls towards their end** (Section 2). **Third, the doubling time measures not speed but time remaining——at 0.4 per hour it is 1.732868 hours, and that is precisely the grace between noticing and running out** (Section 2). **Fourth, the slowing does not begin until the resource is nearly gone——on the Monod relation with a half-saturation constant of 0.5 g/L, the rate is 0.975610 of the maximum at 20 g/L and still 0.909091 at 5 g/L** (Section 3). **Fifth, it halves only when the substrate reaches the half-saturation constant——exactly 0.500000 at 0.5 g/L and 0.166667 at 0.1 g/L. By the time a warning appears there is nothing left** (Section 3). **Sixth, the separator is whether the limit is oneself or the resource——while one's own capacity limits, growth is exponential; when the resource limits, it passes to the Monod form, and by then it is over** (Section 4). **Keywords**: exponential growth, doubling time, Monod relation, half-saturation constant, batch culture, substrate limitation.

Scope of this paper (scope note): No new theorem or law is claimed——exponential growth, the doubling time and the Monod relation are all standard (Monod 1949, Bailey and Ollis 1986, Doran 2013).Particular fermentations are not discussed——brewing, baking and pharmaceutical processes, the choice of strain and the design of media are not the subject.Death and stationary phases are not treated——stopping through toxic metabolites or falling pH is left out, and exhaustion of the substrate is taken as the end.Representative values are used——a specific growth rate of 0.4 per hour, a yield of 0.5 and a half-saturation constant of 0.5 g/L are representative and move with strain and conditions.Continuous culture is not entered——chemostat theory, dilution rate and washout are not treated. Only a single batch is examined.Relation to earlier papers: Paper 289 showed that the limit of forecasting is set by one number, the doubling time of the error, and counted how few days are bought by making observations ten times finer ——there what doubled was an error and the doubling time fixed how far ahead one could see; here what doubles is a population and the same number fixes when it ends.What is added is obtaining the time to exhaustion from the balance, putting the share of the last doubling at **0.505000**, showing that it occupies only **0.150190** of the time, tabulating how late the slowing begins on the Monod relation, and placing the separator on whether the limit is oneself or the resource.

1. Notation and the Question

Setting: cells X grow at a specific rate μ and consume substrate S with yield Y. One culture runs until the substrate is exhausted.

$$\frac{dX}{dt} = \mu X, \qquad t_d = \frac{\ln 2}{\mu}, \qquad X_{\max} = X_0 + Y S_0, \qquad t_{\text{end}} = \frac{1}{\mu} \ln \frac{X_{\max}}{X_0}$$

Take $\mu = 0.4$ per hour, $X_0 = 0.1$ g/L, $S_0 = 20$ g/L and $Y = 0.5$.

Question: what exactly is dangerous about exponential growth: the speed, or the division?

2. Half of It Sits in the Last Doubling

Computation 2.1: the doubling time is 1.732868 hours, the final cell concentration 10.1 g/L, and the substrate lasts 11.537801 hours.

Computation 2.2: taking the cell concentration at successive fractions of its final value gives the time and the substrate remaining.

time	cells	fraction of substrate left
0.0000 hours	0.100000 g/L	1.000000
6.3392 hours	1.262500 g/L	0.883750
8.0721 hours	2.525000 g/L	0.757500
9.8049 hours	5.050000 g/L	0.505000
11.5378 hours	10.100000 g/L	0.000000

The difference between the last row and the one before it is the last doubling.

> **Observation 2.3**
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> **this is the core: 0.150190 of the time consumes 0.505000 of the substrate.Everything before it, 9.804933 hours, consumes 0.495000——at the moment it still looks half full, only one doubling remains.**

> **Observation 2.4**
>
> **this is not a statement about speed.Lowering μ lengthens everything but does not change the share taken by the last doubling——the ratio does not contain μ; it follows from the exponential form alone.**

> **Observation 2.5**
>
> **the doubling time is a measure of time remaining, not of speed.The grace between noticing something wrong and running out is about one doubling, here 1.732868 hours——setting the interval between checks longer than the doubling time means missing half of it.**

3. The Slowing Comes Late

Computation 3.1: on the Monod relation with a half-saturation constant of 0.5 g/L, the specific growth rate was derived for each amount of substrate left.

substrate	specific growth rate	against the maximum
20 g/L	0.390244 per hour	0.975610
5 g/L	0.363636 per hour	0.909091
1 g/L	0.266667 per hour	0.666667
0.5 g/L	0.200000 per hour	0.500000
0.1 g/L	0.066667 per hour	0.166667

Observation 3.2

a fortyfold fall in substrate costs only a factor of 0.909091 in rate.The half-saturation constant is by definition the concentration at which the rate halves, and until that point the slowing is invisible——monitoring the rate raises a warning only once less than 0.5 g/L remains.

Observation 3.3

the two views show the same ending with different faces.In time, half the consumption sits in the last doubling; in concentration, nothing happens until the half-saturation constant is crossed——both say that the end is invisible until just before it.

4. The Separator

Observation 4.1

the separator is whether the limit is oneself or the resource.While one's own capacity to grow limits, growth is exponential; when what is left limits, it passes to the Monod form——and by the time it passes, almost nothing usable is left.

Observation 4.2

beside Paper 289.Paper 289 showed that the limit of forecasting is set by one number, the doubling time of the error——there what doubled was an error and the doubling time fixed how far ahead one could see; here what doubles is a population and the same number fixes when it ends. One number looks forward as a horizon and backward as a grace period.

Observation 4.3

what is not said.Death phase, toxic metabolites and continuous culture are not treated, and every coefficient is representative——what is shown is a division that follows from the exponential form alone.

5. Roll Call and Closing

1. **doubling time 1.732868 hours** (Computation 2.1): **at 0.4 per hour.**
2. **exhaustion at 11.537801 hours** (Computation 2.1): **from the balance.**
3. **last doubling takes 0.505000** (Computation 2.2): **the core.**
4. **that is 0.150190 of the time** (Observation 2.3): **short.**
5. **9.804933 hours give 0.495000** (Observation 2.3): **long and quiet.**
6. **the share has no μ in it** (Observation 2.4): **not about speed.**
7. **0.975610 at 20 g/L** (Computation 3.1): **no slowing.**
8. **0.909091 at 5 g/L** (Computation 3.1): **still invisible.**
9. **0.166667 at 0.1 g/L** (Computation 3.1): **visible too late.**
10. **oneself or the resource** (Observation 4.1): **the separator; Paper 289.**

Closing: **What is frightening about exponential growth is not its speed.Of 11.537801 hours in all, the last doubling of 1.732868 hours, that is 0.150190 of the time, consumes 0.505000 of the substrate.Everything before it, 9.804933 hours, consumes only 0.495000——at the moment it still looks half full, only one doubling remains.This division does not contain the growth rate; it follows from the exponential form alone.Seen from the side of concentration it is the same: on the Monod relation the rate is still 0.909091 of the maximum at 5 g/L, and halves only at the half-saturation constant of 0.5 g/L——by the time the slowing is visible, nothing is left.The separator is whether the limit is one's own capacity to grow or what is left of the resource.Placed among the earlier papers——in Paper 289 the**

doubling time fixed a horizon of forecasting; here the same quantity fixes a grace period.To be honest——death phase and continuous culture are not treated, and every coefficient is representative.

References

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