One Word, “Resonance”, Names Both the Places That Expel Bodies and the Places That Protect Them

— against Jupiter the **3:1**, **5:2**, **7:3** and **2:1** orbits are all empty gaps, yet **3:2** holds the Hilda group and **1:1** the Trojans — the separator is whether a protecting mechanism operates — [Paper 687]

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— *From thinking to syncing.* —

Abstract

Gaps are explained by resonance. The protection of bodies is explained by resonance too.That one commensurability carries opposite consequences is set out here in distances.No new theorem or law is claimed. First, and this is the core. One word names both the emptying and the holding——3:1 against Jupiter, at 2.501245 AU, is a gap, while 3:2, at 3.970480 AU, holds the Hilda group (Section 2). **Second, the gaps are not one but four in a row——2.501245 AU for 3:1, 2.824514 for 5:2, 2.957463 for 7:3 and 3.277559 for 2:1** (Section 2). **Third, the holding side is not one either——the Hilda group at 3:2 and the Trojans at 1:1, at 5.202800 AU; all four gaps lie inside 3:2** (Section 2). **Fourth, the holding side has an instance further out as well——3:2 against Neptune falls at 39.402847 AU, only 0.077153 AU from Pluto’s 39.48 AU** (Section 3). **Fifth, the distance follows from the period ratio in one line——the map is the 2/3 power: twice the period gives 1.587401 times the distance and three times gives 2.080084** (Section 3). **Sixth, the separator is whether a protecting mechanism operates——a commensurability carries no direction of its own; what empties or holds is whether close approaches are avoided under it** (Section 4). **Keywords**: orbital resonance, Kirkwood gaps, Hilda group, Trojan asteroids, Kepler’s third law, two opposite consequences.

Scope of this paper (scope note): **No new theorem or law is claimed——the Kirkwood gaps, the Hilda and Trojan groups and the 3:2 resonance of Neptune and Pluto are all standard (Murray and Dermott 1999, Morbidelli 2002).The dynamics is not derived——perturbation calculations for mean-motion resonances, Lyapunov times and resonance widths are not treated. Only the $2/3$ power that gives the distance is used.The protecting mechanism is not shown——that close approaches are avoided at 3:2 while eccentricity grows at 3:1 is taken as given and not derived here.Individual bodies are not treated——no asteroid is identified and no family is classified.The semi-major axes are not claimed——5.2028 AU for Jupiter and 30.07 AU for Neptune are representative and move with eccentricity and perturbation.Relation to earlier papers**: **Paper 337 showed the solar system to have five edges differing by a factor of 5106——there one word named several places; here one word names opposite consequences. What differs is not a magnitude but a sign.**What is added is **setting gaps and populations in one table**, **showing all four gaps to lie inside 3:2**, **showing 3:2 against Neptune to match Pluto within $\mathbf{0.077153}$ AU**, **putting the map from period ratio to distance at the $2/3$ power**, and **placing the separator on whether a protecting mechanism operates**.

1. Notation and the Question

Setting: **write a_p for the semi-major axis of a planet. An orbit whose period stands to the planet's as q to p follows from Kepler's third law in one line.**

$$\frac{T}{T_p} = \frac{q}{p}, \qquad a = a_p\left(\frac{q}{p}\right)^{2/3}, \qquad \frac{a}{a_p} = \left(\frac{T}{T_p}\right)^{2/3}$$

Take $a_p = 5.2028$ AU for Jupiter and $a_p = 30.07$ AU for Neptune.

Question: **why does one commensurability produce both a gap and a population?**

2. Gaps and Populations Alternate

Computation 2.1: **the distance was derived for each commensurability with Jupiter, and set beside what is found there.**

ratio	semi-major axis	what is there
3:1	2.501245 AU	Kirkwood gap
5:2	2.824514 AU	Kirkwood gap
7:3	2.957463 AU	Kirkwood gap
2:1	3.277559 AU	Kirkwood gap
3:2	3.970480 AU	Hilda group (populated)
1:1	5.202800 AU	Trojans (populated)

Observation 2.2

this is the core: the same phrase means opposite things in the two halves of the table.Every row is "in a commensurable orbit with Jupiter"; above, nothing is there, and below, bodies have gathered——it is not the size of the ratio nor its simplicity: 3:1 and 3:2 are equally simple.

Observation 2.3

there is an order to the arrangement.All four gaps lie inside 3:2, and the two outer ratios are populated——a direction is visible, expelled within and held without, though showing it as a mechanism lies outside this paper.

3. An Instance Further Out, and the Map of Distances

Computation 3.1: **3:2 against Neptune falls at 39.402847 AU. Pluto's semi-major axis of 39.48 AU differs by 0.077153 AU, a ratio of 0.998046.**

Observation 3.2

the same 3:2 stands on the holding side at two different planets.The Hilda group and Pluto share one ratio at different places around different planets——it looks as though the ratio fixes the consequence; but then 3:1 would have to hold as well. The ratio is not what fixes it.

Computation 3.3: **the map from period ratio to distance ratio is the $2/3$ power: twice the period gives 1.587401, three times gives 2.080084 and five times gives 2.924018.**

> **Observation 3.4**
>
> **the side of distance is not difficult at all.Where each commensurability falls follows from one line of Kepler's third law——what is difficult is not the place but what happens there.**

4. The Separator

> **Observation 4.1**
>
> **the separator is whether a protecting mechanism operates.A commensurability carries no direction of its own. Under the same ratio, a phase locked so that close approaches are avoided protects, and otherwise eccentricity grows and the body is removed——the single word "resonance" hides that difference of sign.**

> **Observation 4.2**
>
> **beside Paper 337.Paper 337 showed the solar system to have five edges differing by a factor of 5106——there the question was how many places one word named; here it is which sign. When one word hides several things, what is hidden need not be a magnitude.**

> **Observation 4.3**
>
> **what is not said.Perturbation calculations and resonance widths are not treated, and the protecting mechanism is taken as given——what is shown is the single fact that one ratio carries opposite consequences.**

5. Roll Call and Closing

1. **3:1 at 2.501245 AU** (Computation 2.1): **a gap; the core.**
2. **5:2 at 2.824514 AU** (Computation 2.1): **a gap.**
3. **7:3 at 2.957463 AU** (Computation 2.1): **a gap.**

4. **2:1 at 3.277559 AU** (Computation 2.1): **a gap.**
5. **3:2 at 3.970480 AU** (Computation 2.1): **the Hilda group.**
6. **1:1 at 5.202800 AU** (Computation 2.1): **the Trojans.**
7. **all gaps inside 3:2** (Observation 2.3): **there is an order.**
8. **Neptune's 3:2 at 39.402847 AU** (Computation 3.1): **the holding side.**
9. **within 0.077153 AU of Pluto** (Computation 3.1): **it matches.**
10. **does protection operate** (Observation 4.1): **the separator; Paper 337.**

Closing: **One word, "resonance", names both the places that expel bodies and the places that protect them.Against Jupiter, 3:1 at 2.501245 AU, 5:2 at 2.824514 AU, 7:3 at 2.957463 AU and 2:1 at 3.277559 AU are all empty, while 3:2 at 3.970480 AU holds the Hilda group and 1:1 holds the Trojans.3:2 against Neptune falls at 39.402847 AU, only 0.077153 AU from Pluto——the same ratio stands on the holding side at two different planets.The distances themselves follow from one line of Kepler's third law, twice the period giving 1.587401 times the distance——what is difficult is not the place but what happens there.The separator is whether a protecting mechanism operates.Placed among the earlier papers——in Paper 337 one word named five places; here one word names two opposite signs.To be honest——perturbation calculations and resonance widths are not treated, and the protecting mechanism is taken as given.**

References

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