

The Dimension Census

— Why **3+1** Dimensions, as an Over-Determination by Independent Roots —
Certifying, by the Method of Paper 58, that the Selections of $D = 4$ Are Distinct

Author: Yuuki YAMAGISHI (山岸 勇輝)

Affiliation: Yamagishi Complexity Torus Lab, Japan

ORCID: 0009-0006-8587-3450 | **Paper 69** | **Zenodo:** 10.5281/zenodo.21650011

Category: Mathematical Physics / Dimensionality / Structural Interpretation

Keywords: dimensionality, anthropic principle, over-determination, distinct roots, conformal invariance, graviton degrees of freedom, Euler characteristic

— *From thinking to syncing.* —

Positioning of this paper

This paper offers a **structural interpretation** and a systematic compilation of established facts. It does not prove that spacetime must be **4-dimensional** and claims no new physics. It sharpens Paper 64 (special dimensions are *distributed*) into a statement of **over-determination**: at $D = 4$, several criteria select the dimension for *demonstrably independent* reasons, certified by the distinct-roots method of Paper 58. Two criteria that post-date Paper 64 are added: the conformal (Weyl) invariance of the free Maxwell action, which holds only in $D = 4$, and the topological anchor $\chi(S^n) = 1 + (-1)^n$ of Paper 67. No claim contradicting established theory is made.

ABSTRACT

Paper 64 showed that the "special" dimension is not unique but distributed over $\{3, 4, 5\}$. This paper sharpens that at $D = 4$. We assemble a **census**: a systematic accounting of dimension-selecting criteria together with, for each, its mathematical *root*. Space ($d = 3$) is selected by stability (Bertrand; Ehrenfest) and sharp wave propagation (Huygens); capacity ($d = 5$) by the unit-ball volume peak (Paper 59); spacetime ($D = 4$) by *five* criteria: the conformal invariance of the free Maxwell action (only at $D = 4$), the onset of a propagating graviton $N(D) = \frac{1}{2}D(D-3)$, the exotic smooth structures of $\mathbb{R}^4$ (Donaldson) and the Hopf/instanton content, the unique crossing of the trace-reversal factor with the ladder factor (Paper 58), and the even-dimensional anchor $\chi(S^{2n}) = 2$ (Paper 67). The central result **certifies** (Proposition 3.1), by the method of Paper 58, that these five roots are pairwise *independent*—conformal weight, little-group representation theory, differential topology, the GR trace, and the Euler characteristic have distinct origins, none reducible to another. Hence $D = 4$ is not special for a single reason, nor loosely distributed, but **over-determined by independent roots**. Over-determination is descriptive, not a proof of necessity, and the choice of criteria is a modelling choice.

1. Introduction

Arguments for why space is three-dimensional usually exhibit *one* property special to $d = 3$. Paper 64 changed the register: it tabulated several criteria and observed that their optima are *distributed* over $\{3, 4, 5\}$, so that no single "magic dimension" exists.

This paper sharpens the picture at the physically realised value. The observation is that $D = 4$ (spacetime) is selected not once but *several* times, and—the point—for reasons whose mathematical origins are *independent*. Borrowing the discipline of Paper 58 (compare quantities by their origins, not their values), we certify that the several selections of $D = 4$ are *distinct roots*. The result is stronger than any single "why four dimensions" argument: $D = 4$ is **over-determined**. Two criteria unavailable to Paper 64 are included: the conformal (Weyl) invariance of electromagnetism, and the topological anchor $\chi(S^{2n}) = 2$ of Paper 67.

What this paper does not do. It does not prove $D = 4$ is necessary (over-determination is descriptive, not modal). It adds no physics; every criterion is established.

2. The Census

d is the spatial dimension, $D = d + 1$ the spacetime dimension. The table lists the criteria, the dimension each selects, and its mathematical root.

Criterion	Selects	Root (mathematical origin)
Stable orbits / atoms (Bertrand; Ehrenfest)	$d = 3$	effective-potential stability of $r^{-(d-1)}$ forces
Sharp wave propagation (Huygens)	odd d , min. 3	fundamental solution of the wave equation
Maxwell conformal (Weyl) invariance	$D = 4$	conformal weight of the 2-form F : $\int F \wedge \star F$ Weyl-invariant iff $D = 2 \cdot 2$
Propagating graviton $N = \frac{1}{2}D(D-3)$	$D = 4$ onset	little-group representation theory
Exotic $\mathbb{R}^4$; Hopf / instanton	$d = 4$	differential topology (Donaldson; $S^3 \rightarrow S^2$)
Trace-reversal = ladder factor (Paper 58)	$D = 4$	GR trace $(D-2)/(D-3)$ vs. ladder 2
Topological anchor $\chi(S^{2n}) = 2$ (Paper 67)	even D	Euler characteristic $\chi(S^n) = 1 + (-1)^n$
Unit-ball volume peak (Paper 59)	$d = 5$	$V_n = \pi^{n/2}/\Gamma(\frac{n}{2} + 1)$ maximal at 5

On the Maxwell-conformal criterion. The free electromagnetic action $S = -\frac{1}{4} \int F_{\mu\nu} F^{\mu\nu} \sqrt{-g} d^D x$ is invariant under Weyl rescalings $g \rightarrow \Omega^2 g$ if and only if $D = 4$, because the field strength is a 2-form and $\int F \wedge \star F$ is conformally invariant exactly when the form degree is $D/2$ (Weyl, 1918; standard).

On the topological anchor. By Paper 67, $\int_{S^2} K dA = 2\pi\chi(S^2) = 4\pi$; generally $\chi(S^n) = 1 + (-1)^n$ is 2 for even n , 0 for odd n ; the Gauss–Bonnet/Chern anchor exists only in even dimensions, of which $D = 4$ is the first physical case.

3. Certification: the Selections of $D = 4$ Are Distinct Roots

The census would be empty if its several $D = 4$ entries were one argument in disguise. The discipline of Paper 58—do not identify quantities that coincide in value but differ in origin—here works in reverse: we certify the $D = 4$ selections have *different* origins.

Proposition 3.1 (Independence of the $D = 4$ roots)

The five criteria selecting $D = 4$ arise from pairwise distinct mathematical structures:

1. **Conformal weight.** Maxwell invariance follows from $D = 2p$ for a p -form field strength ($p = 2$). A statement about conformal rescaling of a form.
2. **Representation theory.** $N = \frac{1}{2}D(D-3)$ is the dimension of the symmetric traceless rep of the little group $SO(D-2)$. A statement about group representations.
3. **Differential topology.** Exotic $\mathbb{R}^4$ and Hopf/instanton follow from gauge-theoretic invariants of smooth 4-manifolds (Donaldson). A statement about smooth structure.
4. **The GR trace.** $(D-2)/(D-3)$ is the trace-reversal of the Einstein tensor (Paper 58). A statement about metric-trace algebra.
5. **The Euler characteristic.** $\chi(S^{2n}) = 2$ is a topological invariant (Paper 67). A statement about homology.

None is derivable from another as a matter of the mathematics it invokes; they are distinct roots that happen to select the same dimension.

Remark on the proof. Each item cites a self-contained classical result whose derivation uses only the structure named (conformal geometry; representation theory of $SO(D-2)$; smooth 4-manifold theory; tensor trace algebra; homology). Since the derivations share no common premise beyond the value $D = 4$ itself, the roots are independent in the sense of Paper 58. ■

Observation 3.2 (Over-determination)

$D = 4$ is selected by at least five criteria whose roots are independent (Proposition 3.1). Its specialness is *over-determined*: not a single argument, not merely one point of a loose distribution, but the confluence of several mutually irreducible reasons. The same census places $d = 3$ at the confluence of stability and sharp propagation, and $d = 5$ at volume capacity.

4. Reading: Over-Determination Refines the Anthropic Picture

Paper 64 read the distribution as a refinement of the anthropic principle (**3+1** special by a cluster of optima, not a single cause). This census refines it once more: the cluster is not loose; at $D = 4$ it is a *confluence of independent roots*, an over-determination. This is the strongest form the descriptive statement can take without becoming a claim of necessity:

***3+1** is special not because one condition forbids all else, and not merely because several optima lie nearby, but because the physical dimension is over-determined—selected repeatedly, for independent reasons.*

Over-determination is familiar as a signature of structure rather than accident: a value fixed by one equation might be a coincidence; a value fixed independently by five is not. But (Section 5) this remains a descriptive signature, not a proof.

5. Scope of the Claims

What is established

1. The census collects established criteria with their selected dimensions and roots, adding the Maxwell-conformal criterion and the topological anchor of Paper 67 to Paper 64's list.
2. The five $D = 4$ selections are distinct roots (Proposition 3.1); hence $D = 4$ is over-determined (Observation 3.2).

What is not established / Open problem

- That $D = 4$ is *necessary*. Over-determination is descriptive; it strengthens but does not close the anthropic reading.
- Criterion-independence of the conclusion. Which criteria enter is a modelling choice; what is robust is that the included $D = 4$ criteria have independent origins.
- Any new physics (every row cites an established result).
- *Open*: could a single deeper structure unify the five $D = 4$ roots—making the over-determination one cause? If so the census becomes a theorem; if not, the independence of the roots is itself the fact.

6. Conclusion

The dimension census accounts for the criteria selecting $\mathbf{d} = \mathbf{3}$ (stability, sharp waves), $\mathbf{D} = \mathbf{4}$ (Maxwell conformal invariance, graviton onset, exotic/Hopf topology, the factor-two crossing, the even-dimensional Euler anchor), and $\mathbf{d} = \mathbf{5}$ (volume capacity), each with its mathematical root. Applying the distinct-roots method of Paper 58 in reverse, we certify that the five selections of $\mathbf{D} = \mathbf{4}$ are independent. The physical dimension is therefore *over-determined*: special not by one reason, nor by a loose cluster, but by a confluence of mutually irreducible causes. This is a descriptive strengthening of the anthropic reading, not a proof of necessity; whether the independent roots hide a single deeper cause is the governing open problem.

References

1. Ehrenfest, P. (1917). *In what way does it become manifest ... that space has three dimensions?* Proc. Amsterdam Acad., 20, 200–209.
 2. Bertrand, J. (1873). *Théorème relatif au mouvement d'un point attiré vers un centre fixe*. C. R. Acad. Sci., 77, 849–853.
 3. Weyl, H. (1918). *Gravitation und Elektrizität*. Sitzungsber. Preuss. Akad. Wiss., 465–480.
 4. Courant, R. & Hilbert, D. (1962). *Methods of Mathematical Physics, Vol. II*. Interscience.
 5. Deser, S., Jackiw, R. & 't Hooft, G. (1984). *Three-dimensional Einstein gravity*. Ann. Phys., 152, 220–235.
 6. Donaldson, S. K. (1983). *An application of gauge theory to four-dimensional topology*. J. Diff. Geom., 18, 279–315.
 7. Tegmark, M. (1997). *On the dimensionality of spacetime*. Class. Quantum Grav., 14, L69–L75.
 8. [Yamagishi 2026, Paper 58] *The Factor Two as a Distinct Root*. Zenodo. [10.5281/zenodo.21597747](https://zenodo.org/record/105281/1/zenodo.21597747)
 9. [Yamagishi 2026, Paper 59] *The Unit-Ball Volume Peaks in Five Dimensions*. Zenodo. [10.5281/zenodo.21597865](https://zenodo.org/record/105281/1/zenodo.21597865)
 10. [Yamagishi 2026, Paper 64] *The Special-Dimension Distribution Theorem*. Zenodo. [10.5281/zenodo.21616221](https://zenodo.org/record/105281/1/zenodo.21616221)
 11. [Yamagishi 2026, Paper 67] *The Euler-Characteristic Origin of the 4π Patch*. Zenodo. [10.5281/zenodo.21642170](https://zenodo.org/record/105281/1/zenodo.21642170)
-

— *From thinking to syncing.* —

Yuuki YAMAGISHI (山岸 勇輝)

© 2026 Yamagishi Complexity Torus Lab, Japan · Licensed under Creative Commons Attribution 4.0 International (CC BY 4.0)