Having a Period and Having a Clock Are Two Different Things

— A delayed oscillator with $a = 1$, $b = 2$, $c = 1$ and a lag of 1 sustains itself at a period 3.418367 times the lag and an amplitude of 1.410224, and starting from 0.1 or from 0.9 lands on the same values — the separator is whether an outside clock fixes the phase — [Paper 691]

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— *From thinking to syncing.* —

Abstract

Tides are tabulated a century ahead while El Nino is uncertain half a year out. Both have a period.That having a period is not the same as being predictable is counted here in the wandering of a phase.No new theorem or law is claimed. First, a self-excited oscillation fixes its period and amplitude perfectly well——the delayed oscillator runs at 3.418367 times the lag with amplitude 1.410224, reached from either 0.1 or 0.9 (Section 2). **Second, and this is the core. What is not fixed is the phase——under noise of one size the spread in phase grows from 0.269803 rad after one period to 1.107469 rad after sixteen** (Section 3). **Third, it grows as $\sqrt{k}$——sixteen periods give 4.104739 times one period, close to $\sqrt{16} = 4$: the phase is diffusing, not drifting** (Section 3). **Fourth, an outside clock stops the growth——under the same noise a forced oscillator goes from 0.078447 to 0.076717 rad, a ratio of 0.977952** (Section 3). **Fifth, the value it saturates at has a closed form——for a restoring strength K it is $\sigma/\sqrt{2K}$, and the measured 0.076717 agrees with 0.075000** (Section 3). **Sixth, the separator is whether an outside clock fixes the phase——with a period but nothing holding the phase, the further ahead one looks the less one can say when** (Section 4). **Keywords**: self-excited oscillation, delayed oscillator, phase diffusion, phase locking, predictability, period against phase.

Scope of this paper (scope note): **No new theorem or law is claimed——self-excitation by a delayed oscillator, phase diffusion and phase locking are all standard (Suarez and Schopf 1988, Pikovsky et al. 2001).Real forecasting is not treated——sea-surface temperature indices, actual lead times and forecast skill are not the subject.The coupled physics is not derived——atmosphere-ocean coupling and equatorial wave propagation are not treated. The delayed oscillator is taken as a representative model and its coefficients are representative.Climate change is not discussed——any relation to warming is not treated.The noise is not claimed——it is taken as white and additive, and the character of real noise is not treated.Relation to earlier papers**: **Paper 649 showed that tide tables reach a century ahead not by solving the tide but by taking it apart——there the decomposition worked because each part was tied to the motion of a body in the sky. Here the same kind of periodic phenomenon has no outside clock, so taking it apart leaves the phase undetermined.**What is added is **showing that period and amplitude are independent of the initial condition even for a self-excited oscillation**, **separating out the phase as the one thing left undetermined**, **putting its growth at $\sqrt{k}$, a factor of $\mathbf{4.104739}$**, **showing the forced case to saturate at $\sigma/\sqrt{2K} = \mathbf{0.075000}$**, and **placing the separator on whether an outside clock is present**.

1. Notation and the Question

Setting: **take a model that sustains itself through a delayed negative feedback, and compare the behaviour of the phase when it is free and when it is tied to something outside.**

$$\frac{dT}{dt} = aT - bT(t-\tau) - cT^3, \qquad \frac{d\phi}{dt} = \omega + \sigma\xi, \qquad \frac{d\psi}{dt} = -K\sin\psi + \sigma\xi$$

Take $a = 1, b = 2, c = 1$ and $\tau = 1$. Here ϕ is a free phase and ψ the difference from an outside clock.

Question: **does having a period mean being predictable?**

2. Period and Amplitude Are Fixed Perfectly Well

Computation 2.1: **solving the delayed oscillator gives a period 3.418367 times the lag and an amplitude of 1.410224.**

Computation 2.2: **starting from 0.9 instead of 0.1 gives the same period 3.418367 and the same amplitude 1.410224.**

Observation 2.3

a self-excited oscillation has a shape without being given one.Wherever it starts, it settles to the same period and the same amplitude——“the period is fixed” is, in this sense, true.

Observation 2.4

which is exactly why it looks predictable.If the system fixes both period and amplitude, it seems enough to measure where it is now——and it is that "where it is now" which collapses in the next section.

3. Only the Phase Is Left Undetermined

Computation 3.1: **under noise of one size, the spread in phase was derived after each number of periods.**

elapsed	self-excited (no clock)	forced (tied to a clock)
1 period	**0.269803** rad	**0.078447** rad
4 periods	**0.571739** rad	**0.073427** rad
9 periods	**0.811555** rad	**0.073007** rad
16 periods	**1.107469** rad	**0.076717** rad

Observation 3.2

this is the core: one noise accumulates in one case and not in the other.In the self-excited case sixteen periods give 4.104739 times one period, close to $\sqrt{16} = 4$——the phase is diffusing: it is not drifting systematically, it is walking away because nothing returns it.

Computation 3.3: **in the forced case sixteen periods give 0.977952 times one period and the spread saturates. The closed form $\sigma/\sqrt{2K}$ is 0.075000, agreeing with the measured 0.076717.**

Observation 3.4

an outside clock does not improve accuracy; it stops error from accumulating.So long as something pulls the phase back, the error in phase halts at a value——a century of tides can be tabulated because a century of error does not pile up, not because the equations are exact.

Observation 3.5

hence “the period is right but the date cannot be given”.Asked for the mean interval, one answers 3.418367; asked when the sixteenth peak falls, one can say only within 1.107469 rad, about 0.176259 of a period——knowledge of a period and knowledge of a date are different things.

4. The Separator

Observation 4.1

the separator is whether an outside clock fixes the phase.With one, the error saturates; without one, it grows as $\sqrt{k}$——the same class of periodic phenomena divides on this single point into the predictable and the unpredictable.

Observation 4.2

beside Paper 649.Paper 649 showed that tide tables reach a century ahead not by solving the tide but by taking it apart——there the decomposition worked because each part was tied to the motion of a body in the sky. Here one may take the system apart and find nothing holding the parts. Whether decomposition helps is itself decided by the presence of an outside clock.

Observation 4.3

what is not said.Real forecasting and the coupled physics are not treated, and both model and coefficients are representative——what is shown is that period and amplitude can be fixed while the phase is not.

5. Roll Call and Closing

1. **period 3.418367 times the lag** (Computation 2.1): **fixed by the system.**
2. **amplitude 1.410224** (Computation 2.1): **likewise.**
3. **same from either start** (Computation 2.2): **from 0.1 or 0.9.**
4. **0.269803 rad after one period** (Computation 3.1): **self-excited.**

5. **1.107469 rad after sixteen** (Computation 3.1): **the core.**
6. **a factor of 4.104739** (Observation 3.2): **close to $\sqrt{16}$.**
7. **forced gives 0.078447 rad** (Computation 3.1): **under the same noise.**
8. **still 0.076717 rad at sixteen** (Computation 3.1): **it does not pile up.**
9. **closed form 0.075000** (Computation 3.3): **$\sigma/\sqrt{2K}$.**
10. **an outside clock or none** (Observation 4.1): **the separator; Paper 649.**

Closing: **Having a period and having a clock are two different things.The delayed oscillator sustains itself at 3.418367 times the lag with amplitude 1.410224, reached from either 0.1 or 0.9——both period and amplitude are fixed by the system.Only the phase is left undetermined: under noise of one size the spread grows from 0.269803 rad after one period to 1.107469 rad after sixteen, as $\sqrt{k}$, a factor of 4.104739.A forced oscillator tied to an outside clock goes from 0.078447 to 0.076717 rad and saturates, agreeing with $\sigma/\sqrt{2K} = 0.075000$——an outside clock does not improve accuracy; it stops error from accumulating.So the mean interval can be given and the date of the sixteenth peak cannot.The separator is whether an outside clock fixes the phase.Placed among the earlier papers——in Paper 649 decomposition worked because each part was tied to a body in the sky; here there is nothing to tie the parts to.To be honest——real forecasting and the coupled physics are not treated, and both model and coefficients are representative.**

References

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