The Line Beyond Which a Population Cannot Return Sits Where It Still Looks Plentiful

— For $dN/dt = rN(N/A - 1)(1 - N/K)$ with $r = 0.5$, $A = 100$ and $K = 1000$, the rate is -0.445995 at $N = 99$ and $+0.453995$ at $N = 101$: two individuals reverse the sign — the separator is which side of the line the population is on — [Paper 696]

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— From thinking to syncing. —

Abstract

Extinction is discussed once only a handful are left. But in a population with a threshold, the direction is already settled while the numbers still look ample.What differs on the two sides of that line is counted here in signs and in time.No new theorem or law is claimed. First, and this is the core. Two individuals reverse the sign——the rate is -0.445995 at $N = 99$ and $+0.453995$ at $N = 101$, exchanging decline for growth (Section 2). **Second, the line is nowhere near the brink——$A = 100$ sits at 0.100000 of a carrying capacity of $K = 1000$, a tenth of a full habitat** (Section 2). **Third, below the line every point heads down——-4.095000 at $N = 90$ and -11.875000 at $N = 50$: the lower it goes the faster it falls** (Section 2). **Fourth, run forward, the gap becomes everything——after sixty years the first has fallen to $1.519847555 \times 10^{-9}$ and the second has reached 1000.000000, a gap of 1000.000000 individuals** (Section 3). **Fifth, and there is time to watch it happen——extinction from $N = 99$ takes 60.837000 years, so two individuals become everything over half a century** (Section 3). **Sixth, the separator is which side of the line the population is on——the same species in the same place growing at the same rate ends at 0 or at K according to that alone** (Section 4). **Keywords**: Allee effect, extinction threshold, unstable fixed point, population dynamics, sign reversal, carrying capacity.

Scope of this paper (scope note): **No new theorem or law is claimed——the Allee effect, the extinction threshold of a strong Allee effect and population models carrying a factor $N/A-1$ are all standard (Allee 1931, Courchamp et al. 1999).No real population is fitted——data for particular species are not used; only the behaviour of the model is examined.Mechanism is not derived——difficulty in finding mates, loss of group defence and the like are named but not derived from the model.The coefficients are not claimed——$r=0.5$, $A=100$ and $K=1000$ are representative and move with species and setting; that the line sits at a tenth of K follows from the values assumed.Policy is not discussed——the design of reserves, harvest limits and reintroduction are not the subject.Relation to earlier papers**: **Paper 692 showed that once mutation rate times genome length exceeds one the sequence cannot be held, and counted the fixed order between length and accuracy——there the threshold was a ceiling pressing down; here it is a floor holding up. Both are of the kind "cross it and there is no return", with opposite orientation.**What is added is **showing the sign to reverse across the line at -0.445995 and $+0.453995$**, **showing the line to sit at 0.100000 of K**, **putting the gap after sixty years at 1000.000000 individuals**, **giving the time to extinction as 60.837000 years**, and **placing the separator on which side of the line the population is**.

1. Notation and the Question

Setting: **write the difficulty of growing when few (a strong Allee effect) with a threshold A, against a carrying capacity K.**

$$\frac{dN}{dt}=rN\left(\frac{N}{A}-1\right)\left(1-\frac{N}{K}\right),\qquad r=0.5,\quad A=100,\quad K=1000$$

The fixed points are $N=0$, $N=A$ and $N=K$; A is unstable and 0 and K are stable.

Question: **what separates "still plenty" from "already too late"?**

2. The Two Sides of the Line

Computation 2.1: **the rate was derived at each population.**

population	rate
50	-11.875000
90	-4.095000
99	-0.445995
100	$+0.000000$
101	$+0.453995$
110	$+4.895000$
200	$+80.000000$

Observation 2.2

this is the core: two individuals reverse the sign.The rate is -0.445995 at $N = 99$ and $+0.453995$ at $N = 101$——a difference of 0.020000 in relative terms sends the population to 0 or to K.

Observation 2.3

the line is nowhere near the brink.$A = 100$ sits at 0.100000 of $K = 1000$, a tenth of a full habitat——the line has already been crossed at a point where one can still say “there are a hundred of them”.

Observation 2.4

below the line every point points the same way.-4.095000 at $N = 90$ and -11.875000 at $N = 50$——the decline accelerates as it falls, so by the time it is noticed it is already faster.

3. Running It Forward

Computation 3.1: after sixty years the population starting at 99 has fallen to $1.519847555 \times 10^{-9}$ and the one starting at 101 has reached 1000.000000, a gap of 1000.000000 individuals.

Computation 3.2: extinction from $N = 99$ takes 60.837000 years.

Observation 3.3

slowness is what allows it to be missed.Half a century passes on the way to zero, so no decade taken on its own looks worse than “somewhat fewer”——the sign was settled long ago, and the small rate keeps it from looking irreversible.

Observation 3.4

two individuals become a thousand.The starting gap is 0.020000 in relative terms and the gap after sixty years is 1000.000000 individuals——near the line, an error of measurement is an error of conclusion.

Observation 3.5

so the question in conservation is not “how many are left”.It is “where is the line” and “are we above it”——without the first the second cannot be answered, and a head count alone does not give the direction.

4. The Separator

Observation 4.1

the separator is which side of the line the population is on.The same species in the same place growing at the same rate ends at 0 or at K according to that alone——a quantity that looks continuous carries a discontinuous consequence at one point.

Observation 4.2

beside Paper 692.Paper 692 showed that once mutation rate times genome length exceeds one the sequence cannot be held——there the threshold was a ceiling pressing down; here the line is a floor holding up. Both say “cross it and there is no return”, one as an upper limit against growing too large and one as a lower limit against falling too far.

Observation 4.3

what is not said.Neither real populations nor policy is entered, and the coefficients are representative——what is shown is that the line can sit at a tenth of capacity, where it still looks plentiful.

5. Roll Call and Closing

1. $\mathbf{-0.445995}$ **at** $N = 99$ (Computation 2.1): **falling.**
2. $\mathbf{+0.453995}$ **at** $N = 101$ (Computation 2.1): **rising; the core.**
3. **exactly** 0 **at** $N = 100$ (Computation 2.1): **an unstable fixed point.**
4. **the line is** $\mathbf{0.100000}$ **of** K (Observation 2.3): **not the brink.**
5. $\mathbf{-4.095000}$ **at** $N = 90$ (Computation 2.1): **faster lower down.**
6. $\mathbf{-11.875000}$ **at** $N = 50$ (Computation 2.1): **accelerating.**
7. **gap of** $\mathbf{1000.000000}$ **after sixty years** (Computation 3.1): **it becomes everything.**
8. **extinction in** $\mathbf{60.837000}$ **years** (Computation 3.2): **slow enough to miss.**
9. **starting gap** $\mathbf{0.020000}$ (Observation 3.4): **the width of measurement error.**
10. **which side of the line** (Observation 4.1): **the separator; Paper 692.**

Closing: **The line beyond which a population cannot return sits where it still looks plentiful.The rate is -0.445995 at $N = 99$ and $+0.453995$ at $N = 101$: two individuals reverse the sign and send the population to 0 or to K——and that line, $A = 100$, sits at 0.100000 of a carrying capacity of 1000, a tenth of a full habitat.Below it every point heads down, -4.095000 at $N = 90$ and -11.875000 at $N = 50$, faster the lower it falls.Yet extinction from $N = 99$ takes 60.837000 years, and no decade taken on its own looks worse than "somewhat fewer"——the sign was settled long ago and the small rate hides it.After sixty years the gap is 1000.000000 individuals, the starting difference of 0.020000 having become the whole of it.The separator is which side of the line the population is on.Placed among the earlier papers——in Paper 692 the threshold was a ceiling; here the line is a floor.To be honest——neither real populations nor policy is entered, and every coefficient is representative.**

References

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