What Makes the Tragedy of the Commons Is Not Greed but One Number, the Head Count

— When one unit contributed creates $\mathbf{1.6}$ for the group and is shared among n, at $n = 1$ the individual gains $+0.600000$ and the gap against the group is 0.000000: there is no dilemma — the separator is n — [Paper 697]

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— *From thinking to syncing.* —

Abstract

The tragedy of the commons is told as a story about greed. Yet with the same payoffs, setting the head count to one makes the dilemma disappear.What actually creates it is counted here by moving nothing but the head count.No new theorem or law is claimed. First, and this is the core. At $n = 1$ there is no dilemma——the individual gains $+0.600000$ and the gap against the group is 0.000000, with the payoffs untouched (Section 2). **Second, the moment n becomes two, the sign reverses——the individual now loses -0.200000, and withholding pays** (Section 2). **Third, the gap widens monotonically with n——**0.800000 **at $n = 2$,** $\mathbf{1.440000}$ **at $n = 10$ and** $\mathbf{1.584000}$ **at $n = 100$** (Section 2). **Fourth, the widening has a ceiling——the gap is $b(1 - 1/n)$, so however large n grows it never exceeds** $\mathbf{1.6}$ (Section 2). **Fifth, the condition for it to pay individually is the head count itself——it requires $b > n$, so $n = 100$ demands $b > 100.000000$; and the total forgone grows in proportion to n, $n = 100$ being** $\mathbf{50.000000}$ **times $n = 2$** (Section 3). **Sixth, the separator is n——moving neither the payoffs nor anything about people, moving n alone makes the dilemma appear and disappear** (Section 4). **Keywords**: public goods, tragedy of the commons, marginal per capita return, social dilemma, effect of group size, individual against collective.

Scope of this paper (scope note): **No new theorem or law is claimed——the public-goods game, the marginal per capita return and the tragedy of the commons are all standard (Hardin 1968, Olson 1965).Repetition is not treated——repeated games, reputation, punishment and tit-for-tat are not treated; only the one-shot structure is examined.Experiments are not treated——what fraction of subjects actually contribute is not the subject, and people do not behave as the model says.Institutions are not discussed——regulation, division and common management are not judged, and the successful cases of common management documented by Ostrom are not entered.The value is not claimed——$b = 1.6$ is representative, and the boundaries move with it.Relation to earlier papers**: **Paper 437 showed that each choosing best lands both on the worst——there the payoffs of two players were fixed. Here the head count is made a variable, the dilemma vanishing at $n = 1$ and the gap widening as n grows. The two players of Paper 437 are the smallest member of this family.**What is added is **setting out individual gain and gap for each head count**, **showing the gap to be exactly 0.000000 at $n = 1$**, **showing it never to exceed b**, **showing the condition to be $b > n$ and the total forgone to grow in proportion to n**, and **placing the separator on the head count**.

1. Notation and the Question

Setting: **contributing one unit creates b for the group, shared equally among n. The contributor's own share is b/n and the cost is 1.**

$$\text{individual} = \frac{b}{n} - 1, \qquad \text{group} = b - 1, \qquad \text{gap} = b - \frac{b}{n} = b\left(1 - \frac{1}{n}\right)$$

Take $b = 1.6$. Since $b > 1$, contributing always pays for the group.

Question: **what creates the dilemma: the payoffs, or the head count?**

2. Moving Nothing but n

Computation 2.1: **the share, the individual gain and the gap were derived for each head count.**

head count	share	individual gain	gap
1	1.600000	+0.600000	0.000000
2	0.800000	−0.200000	0.800000
4	0.400000	−0.600000	1.200000
10	0.160000	−0.840000	1.440000
100	0.016000	−0.984000	1.584000

> **Observation 2.2**
>
> this is the core: at $n = 1$ there is no dilemma.With the payoffs and the cost unchanged, setting the head count to one gives an individual gain of $+0.600000$, matching the group——so the dilemma is in neither the payoffs nor the people.

> **Observation 2.3**
>
> by $n = 2$ the sign has already reversed.Since $b/n = 0.800000 < 1$, withholding pays as soon as a second person exists——no crowd is needed; ceasing to be one is enough.

Computation 2.4: the gap is $b(1 - 1/n)$, rising monotonically with n but never exceeding $b = 1.6$; at $n = 100$ it is 1.584000.

> **Observation 2.5**
>
> the rise saturates.Taking n from 10 to 100, a factor of ten, raises the gap only from 1.440000 to 1.584000 ——most of the damage has already happened among few.

3. The Condition, and the Total Forgone

Computation 3.1: contributing pays individually when $b/n > 1$, that is $b > n$: $b > 2.000000$ at $n = 2$ and $b > 100.000000$ at $n = 100$.

Observation 3.2

the requirement tightens in proportion to the head count.For n people, one unit must create more than n for the group before it pays anyone individually——the more people share it, the higher the bar for the same act of cooperation to be worth it.

Computation 3.3: **the total forgone when nobody contributes is $(b-1)n$: 1.200000 at $n=2$, 6.000000 at $n=10$ and 60.000000 at $n=100$, so $n=100$ is 50.000000 times $n=2$.**

Observation 3.4

the loss per person saturates and the total does not.The gap per person stops at 1.6 while the total keeps growing in proportion to n——"it is not much" is true one person at a time and false for the whole.

4. The Separator

Observation 4.1

the separator is the head count n.Moving neither payoffs nor costs nor anything about people, moving n alone makes the dilemma appear and disappear——n is what drives a wedge between what is good for one and what is good for all.

Observation 4.2

beside Paper 437.Paper 437 showed that each choosing best lands both on the worst——there the payoffs of two players were fixed; here the head count is the variable, and those two players become the smallest case in which the dilemma appears.

Observation 4.3

what is not said.Repetition, punishment, experiments and institutions are not treated——what is shown is that one number, the head count, makes the dilemma appear and disappear.

5. Roll Call and Closing

1. **gap $\mathbf{0.000000}$ at $n = 1$** (Computation 2.1): **the core.**
2. **gain $\mathbf{+0.600000}$ at $n = 1$** (Computation 2.1): **matching the group.**
3. **$\mathbf{-0.200000}$ at $n = 2$** (Computation 2.1): **already reversed.**
4. **gap $\mathbf{1.440000}$ at $n = 10$** (Computation 2.1): **widening.**
5. **gap $\mathbf{1.584000}$ at $n = 100$** (Computation 2.1): **never above b.**
6. **tenfold n adds only 0.144000** (Observation 2.5): **it saturates.**
7. **the condition is $b > n$** (Computation 3.1): **proportional to the count.**
8. **$b > \mathbf{100.000000}$ at $n = 100$** (Computation 3.1): **not realistic.**
9. **total is $\mathbf{50.000000}$ times** (Computation 3.3): **this does not saturate.**
10. **the head count n** (Observation 4.1): **the separator; Paper 437.**

Closing: **What makes the tragedy of the commons is not greed but one number, the head count.When one unit contributed creates 1.6 for the group, at $n = 1$ the individual gains $+0.600000$ and the gap against the group is 0.000000: there is no dilemma——and nothing in the payoffs, the costs or the people has been changed.At $n = 2$ the individual loses -0.200000 and the sign has already reversed; the gap reaches 1.440000 at $n = 10$ and 1.584000 at $n = 100$.The gap is $b(1 - 1/n)$, so it never exceeds 1.6, and taking n from 10 to 100 adds only 0.144000——most of the damage has already happened among few.Making it pay individually requires $b > n$, so $n = 100$ demands $b > 100.000000$, while the total forgone grows in proportion to n and $n = 100$ is 50.000000 times $n = 2$——the loss per person saturates and the total does not.The separator is the head count n.Placed among the earlier papers——Paper 437 fixed the payoffs of two players; here the head count is the variable and those two become one case in a family.To be honest——repetition, punishment, experiments and institutions are not treated.**

References

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